Springdale First School

Imagine, Believe, Achieve

Maths Calculation Policy

Intent:

At Springdale First School we aim to provide high-quality mathematics education; to provide depth, variety and enjoyment each day in every class; and to provide the optimum environment for all children to succeed.

We aim to deliver consistency in pedagogy: it is crucial for children to develop procedural fluency alongside a deep grasp of the base ten number system which stems from all staff teaching the right thing at the right time.

We aim to enable aspiration: secure mathematical skills underpin success in everyday life and facilitate the pursuit of fulfilling lives and well-paid careers.

Context:

The aims of the National Curriculum (2014) states that all children should progress at broadly the same rate whilst developing fluency in fundamental number work, reasoning abilities (both in terms of language and mathematical proof) and the capacity to solve problems (demonstrating flexible and creative thinking in a range of situations).

Mastery:

Maths is a mastery subject. At Springdale this means we believe that all children are capable of understanding and doing mathematics, given the appropriate resources and sufficient time.

Children need to build solid foundations so that their mathematical ability broadens and deepens over time: this requires looking at concepts in detail using a variety of representations and contexts and committing key facts, such as number bonds and times tables, to memory.

Opportunities to acquire and practise mathematical knowledge need to be carefully designed to deliver rich experiences within which calculation algorithms are taught consistently throughout the school and across year groups. These then need to be constantly revisited to ensure they are committed to long term memory and can be easily recalled when needed.

Implementation:

At Springdale, Maths lessons are fun, practical learning opportunities that take place both inside and outside the classroom. They involve lots of discussion and investigation before learning and practising and then moving onto applying and deepening.

Children are taught reasoning and problem solving skills to ensure an in-depth understanding of each concept.

Within our curriculum, time is prioritised to improve number sense and arithmetic proficiency. After developing fluency, children need to show that they can apply their knowledge in mathematics. Therefore, for all maths concepts, teachers need to ensure that children are “Challenged through being offered rich and sophisticated problems.”

We aim to do this through the development of conceptual and procedural variation:

- Procedural variation provides step-by-step intelligent practice, looking at what has stayed the same and what has changed. Certain aspects vary while others are kept constant. The ability to recall facts and manipulate them to work out other facts is important so it involves practice to spot relationships and make connections.
- Conceptual variation explores what something is and isn't, and what's the same or different. This time, instead of varying the problem, the representation of the problem is varied. This is generally through the use of models or images, as well as application across a range of contexts (e.g. time, money, measurement).

Planning

We believe that students who have a good grasp of number make better mathematicians. At Springdale, the Maths curriculum is carefully planned, with opportunities for interweaving of new content and spaced retrieval of taught facts. We follow the NCETM's guidance for teaching Maths NCETM Teaching for mastery. The DfE's 'Ready to progress' documents statements (DfE- Teaching mathematics in primary schools) enable teachers to ensure all children progress through the National Curriculum. Maths at Springdale is taught in small, well sequenced steps. NTS end of term assessments inform our planning and identify any areas of number work that need greater attention and revisiting. EYFS follow the early principles for counting (see page 5) and NCETM's 'Mastering Number' programme (NCETM Mastering Number) to develop the children's early number sense.

Knowledge is constantly revised, ensuring taught content goes into the children's long term knowledge. As well as an 'interweaving approach' to the Maths curriculum, '5-a-day' books are used within Years 1 to 4 to revise taught content and strategies. By constantly revisiting these key concepts, students become more secure which mathematical procedures and concepts. We understand that some children need more revisiting of some concepts than others. For this, we have 'Keep Up, Catch Up' sessions with teaching assistants. We also use Precision Teaching to help build speed and fluency.

Key Number Facts

At Springdale First School we recognise the importance of establishing a secure foundation in mental calculations and recall of number facts before standard written methods are introduced. This begins with regular and frequent counting. Students have many opportunities to count forwards and backwards from different starting points to see patterns and understand the mechanics of a base-10 number system.

When counting, students will be exposed to a variety of resources, both physical and online. This includes hundred squares, number lines, number tracks, dienes, rekenreks, numicon, and place value flip chart (online). Discussions around what is happening and why take place and allow students to predict which number will appear next.

Each year group will also count forwards and backwards in different jumps, depending on the knowledge relevant. For example:
- Reception: on and back in ones
- Y1 counting in 2s, 5s and 10s
- Y2 make links with counting in 10s crossing boundaries
- Y3 begin to apply this knowledge to reading scales and fractions (counting in tenths- 0.1, 0.2, 0.3, etc. and halves 0.5, 1.0, 1.5, 2.0, etc. quarters ¼, ½, ¾, 1, 1 ¼ etc.)
- Y4 count using a range of multiples and in a number of ways (e.g. 25s, 250s, 2.5s, 0.25s)

We use materials such as 'TimesTables Rockstars' and 'NumBots' to provide regular practise of key number facts to develop fluency. We aim for students to practise counting, subitising, and times table facts in school at 3-4 times a week and at home we encourage them to practise for 20 minutes per week minimum.

Times tables:
Year 2 – 2, 5 and 10
Year 3 – as above and 3, 4 and 8
Year 4 – all times tables to 12x12

These should be achieved in a minimum of 6 seconds or less.

Alongside this, we have also created a 'Maths Rockstars' programme that focuses on key addition, multiplication, and division facts. These are broken into 'steps'. Students start learning and being tested on each step from the Summer term in Year 1.

When being tested, students are allowed 4 minutes per Maths Rockstar sheet. If they complete the sheet in the allotted time, achieving 100%, then they will progress to the next step.

Year 1 – Achieving to Step 6
Year 2 – Achieving to Step 12
Year 3 – Achieving to Step 18
Year 4 – Achieving to Step 28

Concrete, pictorial, abstract

We begin our maths teaching with practical resources wherever possible. Concrete apparatus supports children with their ability to grasp new concepts. We then move onto pictorial representations and 'bar models' to represent numbers in worded problems, fractions and, later, algebra. This helps our pupils to visualise the relationships between numbers.

Cross-curricular links

Mathematics is mainly taught as a discrete subject but every effort is made to link maths with other areas of the curriculum. We try and identify the mathematical possibilities across the curriculum at the planning stage. We also draw children's attention to the links between maths and other curricular work so that they begin to understand that maths is not an isolated subject. In the Early Years, these links are more evident through play based learning.

Learning environment

We recognise the importance of displays in the teaching and learning of mathematics. Every class displays a working wall which demonstrates examples of today's lesson as well as previous lessons. This is because learning is a continuum and prior learning should be recalled and retrieved before moving onto something new.

Displays will also include key vocabulary for children to call upon when working independently, especially in relation to reasoning questions. Also available should be examples of bar models and strategies for problem-solving (use of tables, drawing pictures, trial and improve etc.). By having these readily available to students, it will foster a greater sense of independence when working.

Subitising

We understand the importance of understanding what number is, 'The Threeness of Three'. Students are constantly taught subitising skills to ensure they gain a solid understanding of what a number is and what it is made up of. This can include the use of numicon, tens frame, dots, playing cards, pictures etc.

Guidance on using this Springdale Maths Calculation Policy

- While these methods are linked to year groups, this should only be as a guide and each year group should assess children's knowledge and understanding of concepts. Methods should be built upon and revisited by every year group to ensure retention of each method.
- Teaching written calculation methods does not replace the need to revisit and use mental strategies. These are the building blocks of solid number work.
- All the images selected should act as guidance and not constraints.
- Children should be encouraged to first approximate their answers before calculating, and then check their answers after calculating using an appropriate strategy.
- Children should be encouraged to consider if a mental calculation would be more appropriate before using a written method.

This policy will ensure consistency and progression in our approach to the learning and teaching of calculations across different year groups. It will enable our children, teachers and parents to work in partnership, developing an efficient, reliable, formal written method of calculation for all operations and to use these methods accurately with confidence for understanding.

It is important to ensure that everyone who uses this policy understands that the methods described under each year group are not specific to that year group. Each method should be recalled, utilised, and built upon in a way that is suited to the calculations and content being taught.

The representations used within this policy are examples from the NCETM Professional Development documents (Mastery Materials | NCETM) and DfE's Ready to Progress documentation Mathematics guidance: key stages 1 and 2 (covers years 1 to 6) (publishing.service.gov.uk)*. They should not be copied or reproduced for further use than is intended within this policy.*

The stable order principle

When counting, the names of numbers stay in the same order.

one, two, three, four, five ✓

one, three, two, five, four ✗

Children at Springdale learn to say the order of numbers through rhymes, songs and stories.

The one to one principle

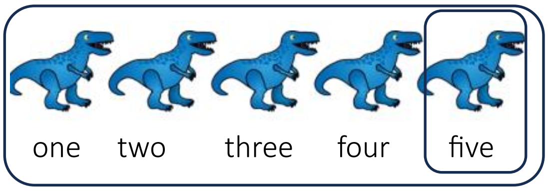
Matching one 'counting word' with each object in a set to be counted. Saying one number word, and only one number word, for each object being counted.

✓

✗

Cardinal principle

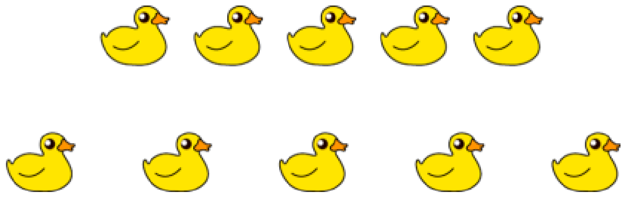
The final number said when counting represents the total number in a group. Children at Springdale are taught that the purpose of counting is to find out how many items there are altogether.



"There are one, two, three, four, five dinosaurs. I have five dinosaurs altogether."

The conservation principle

The number of items in a given group is the same whether the tems are spread far apart or are close together.



The abstract principle

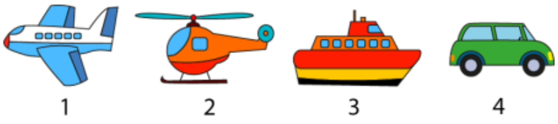
The same principles of cardinality (the final number said is the final number counted) can be applied when counting anything, whether that be large and small objects, objects of different colour or type. Counting can also be applied to any set, not just physical objects. Children start to count tems that they can't see or touch but can hear or imagine. E.g. stomps, hops, claps.

"One, two, three – there were 3 drums beats altogether."

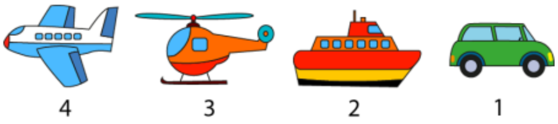
The order irrelevance principle

The total number of objects in a set does not change if you count them in a different order. It doesn't matter which item you start on, the total stays the same.

'How many vehicles are there? Start counting with the aeroplane.'



'How many vehicles are there? Start counting with the car.'



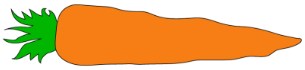
Progression of strategies for Addition

Introduced in EYFS and KS1

1. The use of the language **'part, part and whole'.**

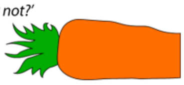
'Part, part and whole' are used throughout addition and subtraction. The language and concept of part and whole are introduced in EYFS and continued into Year 1

'This is a whole carrot, because I have all of it.'



"This is a whole ________ because I have all of it"

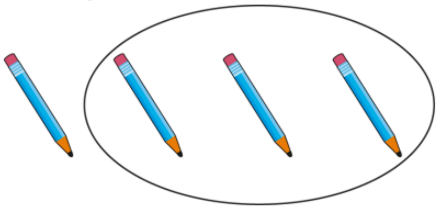
'Is this a whole carrot?'
'Why / why not?'



"This is not a whole _____ because I only have part of it."

2. The language of **'part, part and whole'** when describing a quantity of objects.

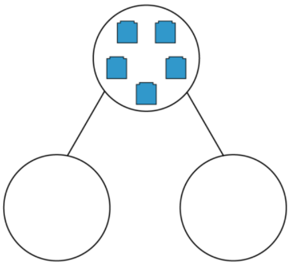
- *'There are four pencils in the whole group.'*
- *'There are three pencils in the part of the group that has a ring around it.'*



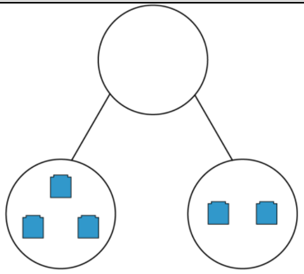
"There are 4 pencils in the whole group.
There are 3 pencils in this part of the group."

3. Introduction of the **'part, part and whole'** model.

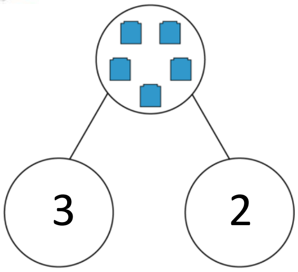
Manipulatives to be introduced – multilink cubes.



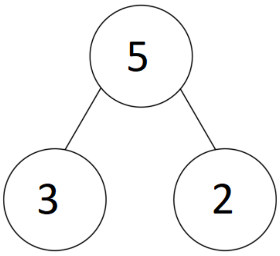
"There are five cubes in the whole group. Five is the whole."



"Three is a part, two is a part."



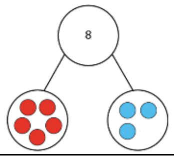
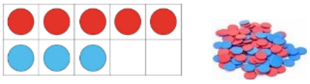
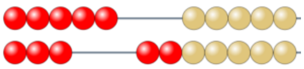
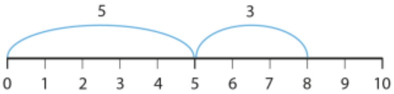
"Three is a part, two is a part. Five is the whole."



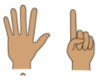
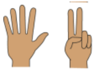
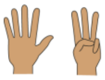
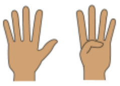
"The 5 represents the whole.
The 3 represents a part.
The 2 represents a part."

4. Composition of the numbers 0 to 10.

Children explore the composition of the numbers 0 to 5 and then 6 to 10, working towards a systematic approach to finding all of the ways each number can be partitioned.
Subitising is used for the facts from 0 to 5 and then the facts for 6 to 10 are taught as '5 and a bit'.
The following manipulatives (resources) are introduced to the children and are used to 'illuminate' the structure of these numbers.

Part, part, whole model:	A tens frame and double sided counters:	Rekenrek:	Numberline:
			

At this stage, the + addition symbol is not yet introduced. The children know that a part and a part combine together to make a whole.

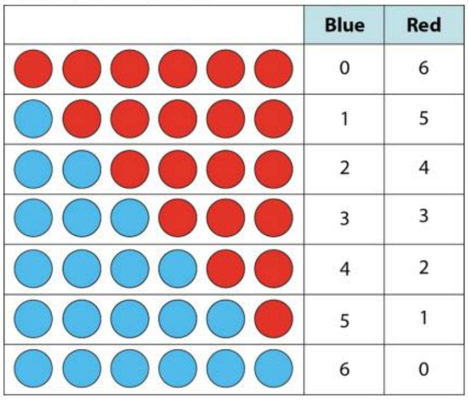
Taught through subitising using the manipulatives above						Taught as 5 and a bit numbers using the manipulatives above plus their fingers as shown below.				
0	1	2	3	4	5	6	7	8	9	10
0 and 0	0 and 1 1 and 0	0 and 2 1 and 1 2 and 0	0 and 3 1 and 2 2 and 1 3 and 0	0 and 4 1 and 3 2 and 2 3 and 1 4 and 0	0 and 5 1 and 4 2 and 3 3 and 2 4 and 1 5 and 0	5 and 1 	5 and 2 	5 and 3 	5 and 4 	5 and 5 

5. Composition of the numbers 6 to 9.

The children are then taught to find the following facts through a systematic approach:

6	7	8	9
0 and 6	0 and 7	0 and 8	0 and 9
1 and 5	1 and 6	1 and 7	1 and 8
2 and 4	2 and 5	2 and 6	2 and 7
3 and 3	3 and 4	3 and 5	3 and 6
4 and 2	4 and 3	4 and 4	4 and 5
5 and 1	5 and 2	5 and 3	5 and 4
6 and 0	6 and 1	6 and 2	6 and 3
	7 and 0	7 and 1	7 and 2
		8 and 0	8 and 1
			9 and 0

'Turn one counter over at a time and name the parts that make the whole number.'



	Blue	Red
🔴🔴🔴🔴🔴🔴	0	6
🔵🔴🔴🔴🔴🔴	1	5
🔵🔵🔴🔴🔴🔴	2	4
🔵🔵🔵🔴🔴🔴	3	3
🔵🔵🔵🔵🔴🔴	4	2
🔵🔵🔵🔵🔵🔴	5	1
🔵🔵🔵🔵🔵🔵	6	0

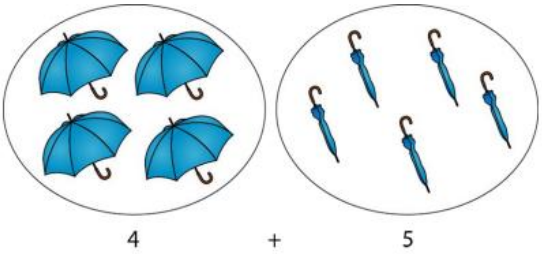
6. Introduction of the + symbol.

Language: addend + addend

+ is referred to as add or plus.

Addition means **putting together.**

'Putting together'

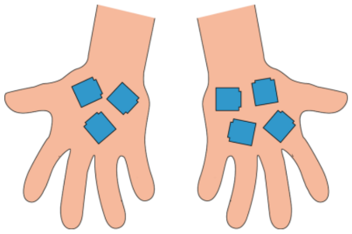


- *'There are four open umbrellas and five closed umbrellas.'*
- *'We can write this as four plus five.'*

  4 + 5

- *'The 4 represents the four open umbrellas.'*
- *'The 5 represents the five closed umbrellas.'*

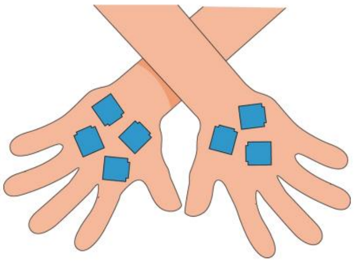
7. The order of the addends is irrelevant, the whole is the same.



'There are three cubes in this hand.' *'There are four cubes in this hand.'*

'We can write this as three plus four.'

3 + 4



'There are four cubes in this hand.' *'There are three cubes in this hand.'*

'We can write this as four plus three.'

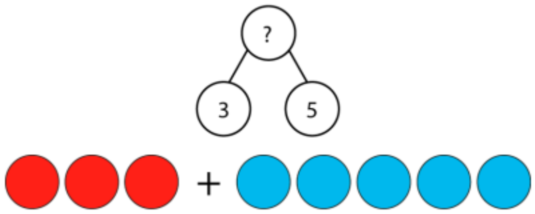
4 + 3

8. Introduction of the = (equal to) symbol.

Children are already familiar with this through their place value work using comparative language (<, > and =).

addend + addend = sum

sum = addend + addend



3 + 5 = 8

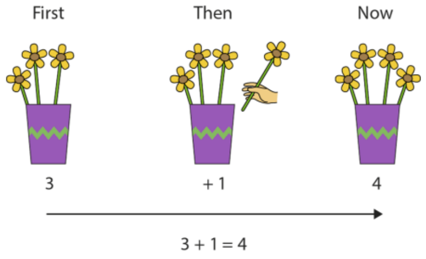
5 + 3 = 8

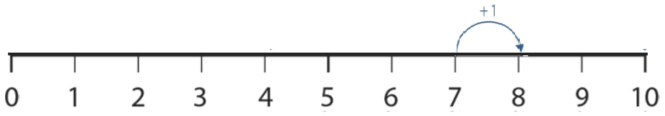
8 = 3 + 5

8 = 5 + 3

9. Adding one gives one more.

Concrete representations will be used to show that, when one more item is added to the whole, the number grows by one, it gives one more. Numberlines will be used predominantly as a written calculation for +1.

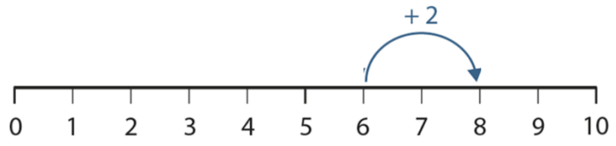


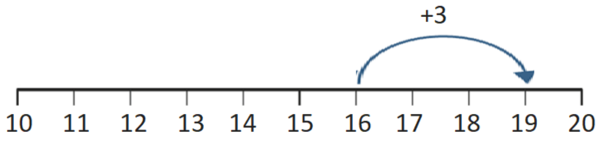


“Adding one gives one more.”

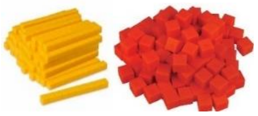
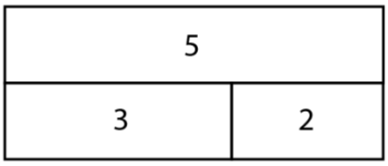
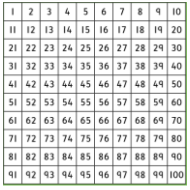
10. Adding 2 or more when the answer does not cross ten using a numberline.

When adding two or more, the following will be shown on a numberline.



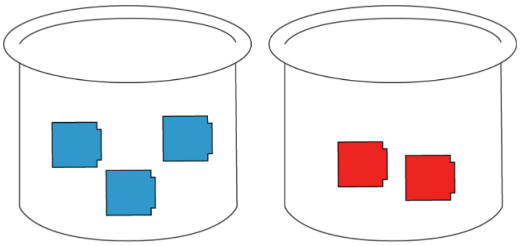


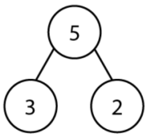
11. Introduction of the following manipulatives (resources):

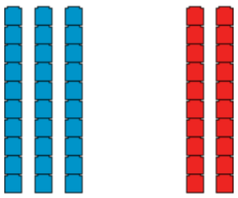
Diennes	Bar model	Bead string to 100	100 squares
			

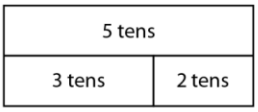
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

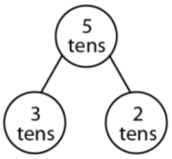
12. Use known facts to deduce others.











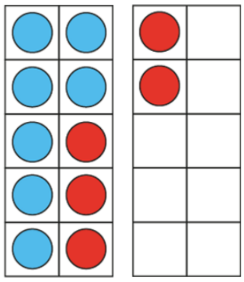
3 tens + 2 tens = 5 tens

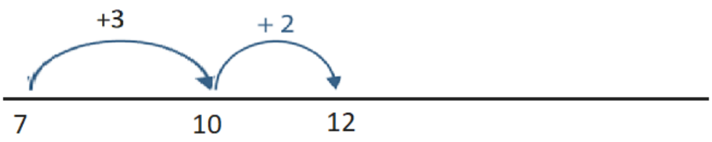
$$3 + 2 = 5$$

$$30 + 20 = 50$$

13. The ‘make ten’ strategy for bridging 10.

7 + 5 =





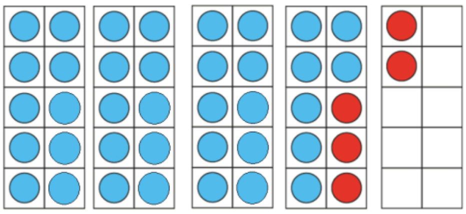
$$7 + \underset{3\quad 2}{5} = \boxed{7 + 3} + 2$$

$$= 10 + 2$$

$$= 12$$

13. The ‘make ten’ strategy for bridging a multiple of 10.

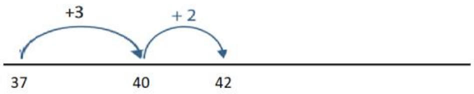
37 + 5 =



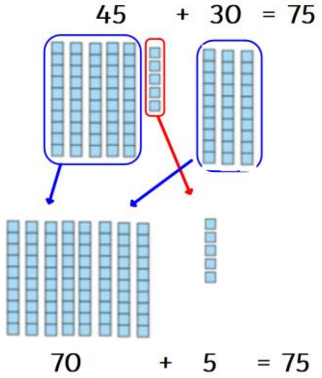
$$37 + \underset{3\quad 2}{5} = \boxed{37 + 3} + 2$$

$$= 40 + 2$$

$$= 42$$



14. Partitioning to add a multiple of 10 onto a 2 digit number.



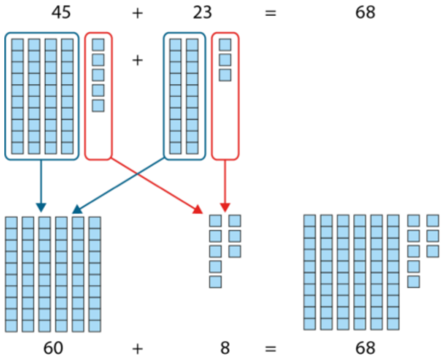
45 + 30 =

40 5

40 + 30 = 70

70 + 5 = 75

15. Partitioning to add two 2 digit numbers (no bridging)



45 + 23

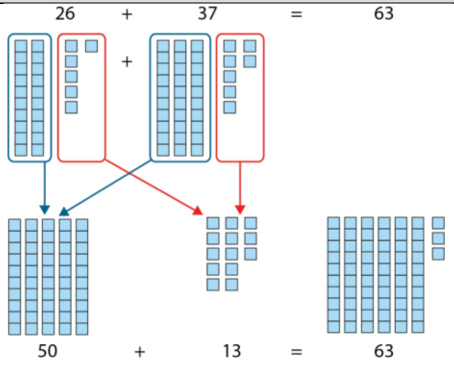
40 5 20 3

40 + 20 = 60

5 + 3 = 8

60 + 8 = 68

16. Partitioning to add two 2 digit numbers (bridging the tens)



26 + 37

20 6 30 7

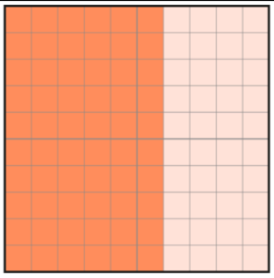
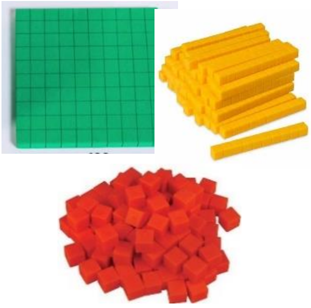
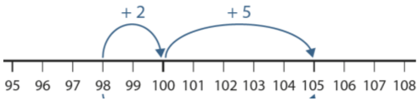
20 + 30 = 50

6 + 7 = 13

50 + 13 = 63

Introduced in KS2

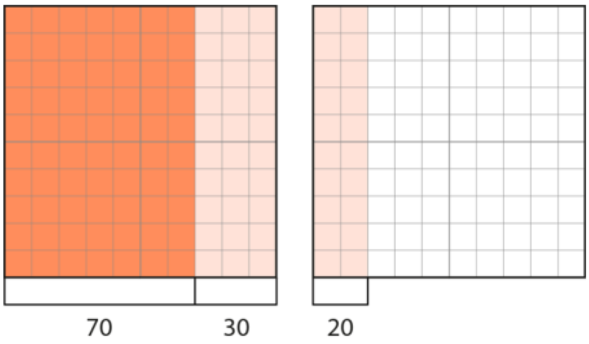
The following manipulatives should be used to 'illuminate' the structure of these calculations.

	Diennes 	Place value counters 	Numberline: 

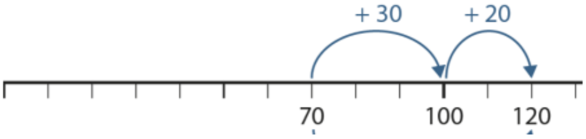
1. Addition across 100 in multiples of ten- bridging

70 + 50 =

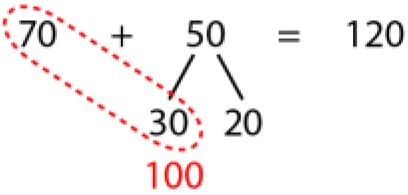
Hundred grids:



Number line:



Equations:

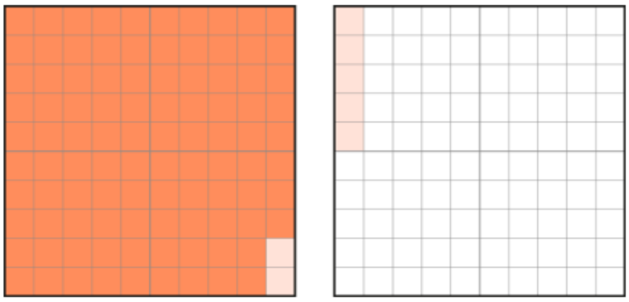


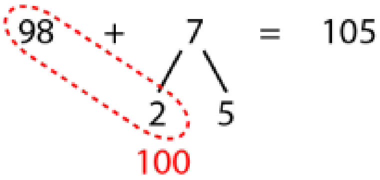
$$70 + 50 = 70 + 30 + 20$$
$$= 100 + 20$$
$$= 120$$

2. Addition across 100 – bridging when adding a single digit number.

98 + 7

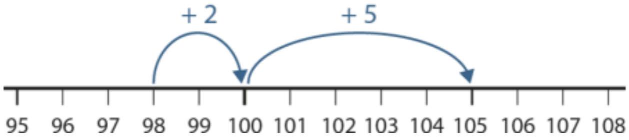
Hundred grids:





$$98 + 7 = 98 + 2 + 5$$
$$= 100 + 5$$
$$= 105$$

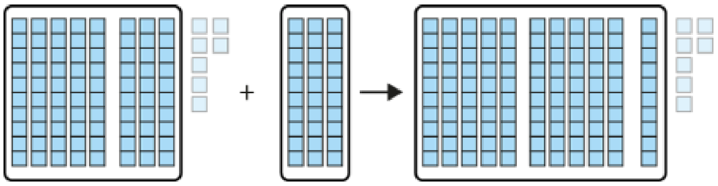
Number line:



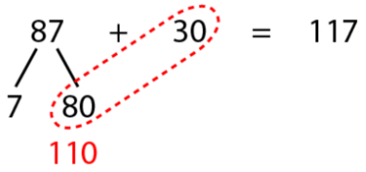
3. Addition across 100 – when adding a multiple of 10.

87 + 30

Dienes:



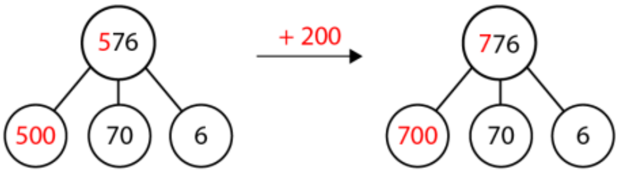
87 + 30 = 117



$$\begin{aligned}87 + 30 &= 80 + 7 + 30\\ &= 110 + 7\\ &= 117\end{aligned}$$

4. Addition and subtraction of 10s and 100s to a 3 digit number.

Addition and subtraction of 100s:

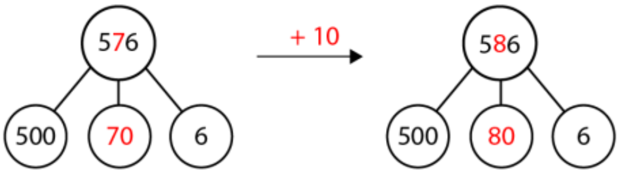


576 + **2**00 = **7**76

Example description – addition:

- *'We had **five** hundreds, then we added **two** hundreds, so now we have **seven** hundreds.'*
- *'**Five** hundred and seventy-six plus **two** hundred is equal to **seven** hundred and seventy-six.'*

Addition and subtraction of tens:

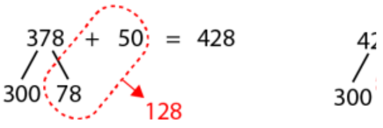


57**6** + **1**0 = 5**8**6

Example description – subtraction:

- *'We had **eight** tens, then we subtracted **one** ten, so now we have **seven** tens.'*
- *'Five hundred and **eighty**-six minus **ten** is equal to five hundred and **seventy**-six.'*

5 Applying knowledge of bridging 100:



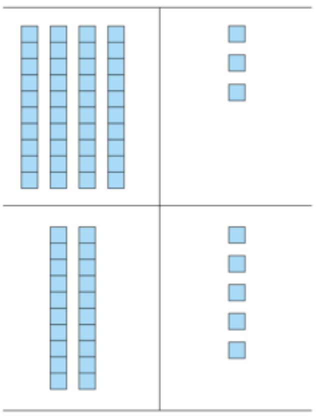
Equations – adding the multiples of ten first:

$$\begin{aligned}378 + 50 &= 370 + 50 + 8\\ &= 420 + 8\\ &= 428\end{aligned}$$

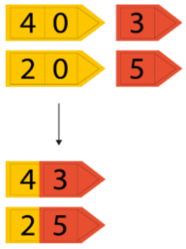
7. A formal method for addition – column addition – no regrouping

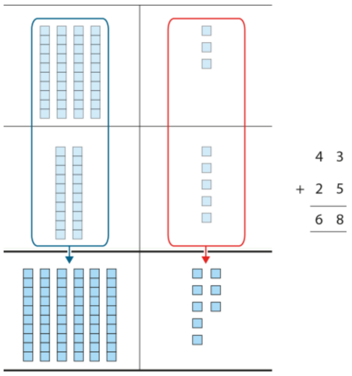
The following manipulatives would be used for additions with 2 digits, 3 digits and 4 digits.

43 + 25 =



Arrow cards:

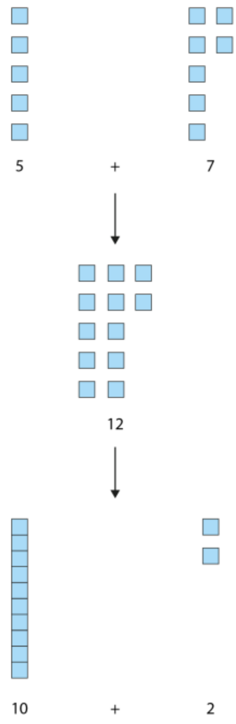




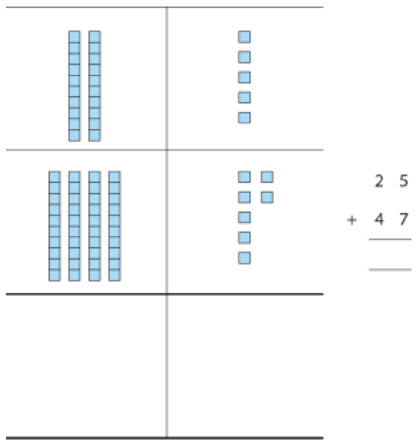
Column addition on squared paper:

	4	3	
+	2	5	

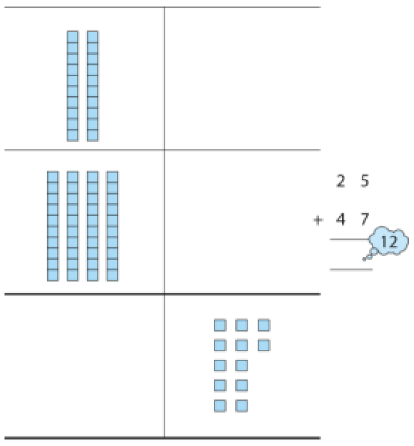
8. A formal method for addition – column addition – with an exchange (exchanging one ten for 10 ones).



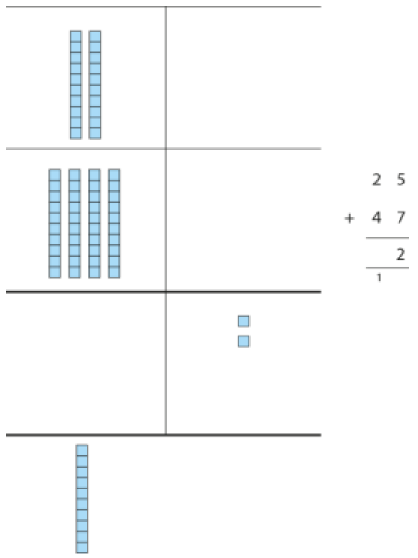
Step 1



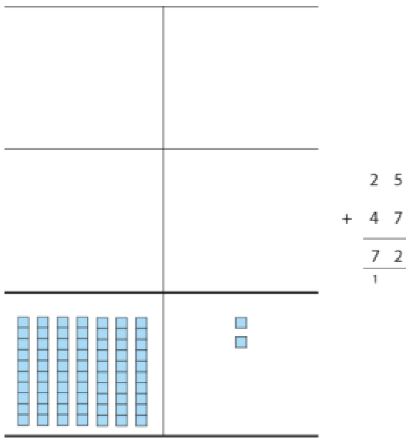
Step 2



Step 3

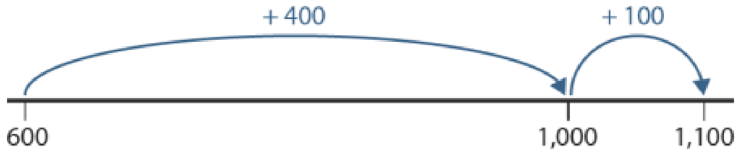


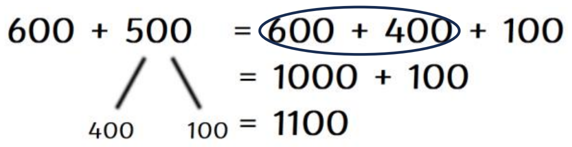
Step 4



9. Addition across 1000 in multiples of hundred- bridging

600 + 500 =



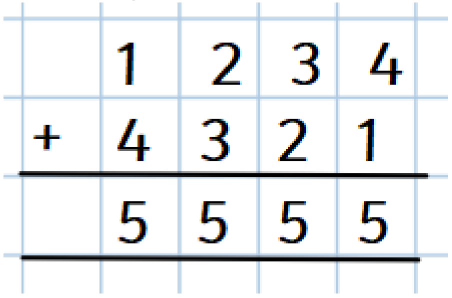


$$600 + 500 = 600 + 400 + 100 = 1000 + 100 = 1100$$

10. Column addition with 4 digit numbers.

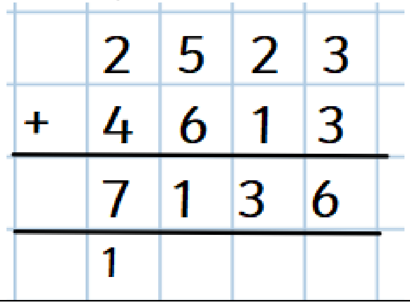
A similar progression of concrete, pictorial, abstract would be followed as in point 8 of KS2 addition.

No exchange



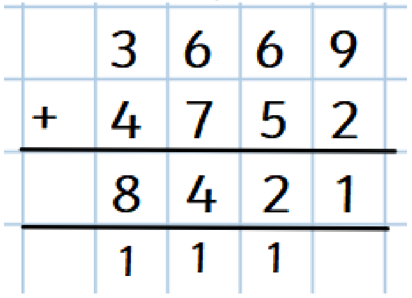
	1	2	3	4
+	4	3	2	1
	5	5	5	5

1 exchange



	2	5	2	3
+	4	6	1	3
	7	1	3	6
	1			

2 or more exchanges



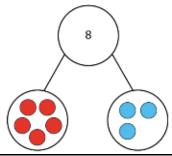
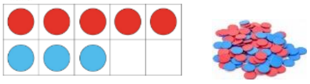
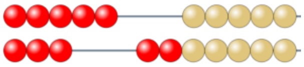
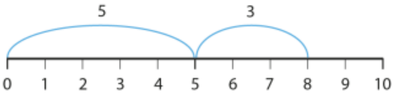
	3	6	6	9
+	4	7	5	2
	8	4	2	1
	1	1	1	

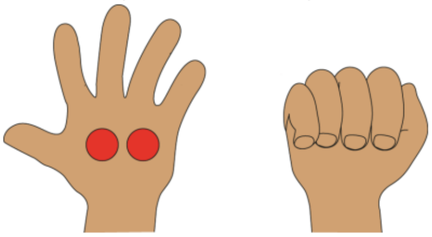
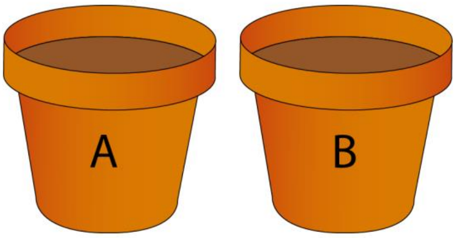
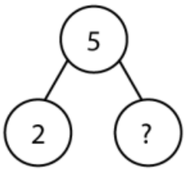
Progression of strategies for Subtraction

Introduced in EYFS and KS1

1. Composition of the numbers 0 to 10.

Whilst exploring the composition of the numbers 0 to 5 and then 6 to 10 as shown in the addition section of this policy, the related subtraction facts will also be illuminated. The children will be introduced to the sentence stem, **"If we know one part, we can find the other part."** At this stage, the- subtraction symbol is not yet introduced.

Part, part, whole model: 	A tens frame and double sided counters: 	Rekenrek: 	Numberline: 

'I have five counters. There are two counters in my open hand. How many counters are there in my closed hand?' 	*'There are five seeds altogether. Four of the seeds are in Pot A. How many are in Pot B?'* 	 The whole is five and one part is two so the other must be three.

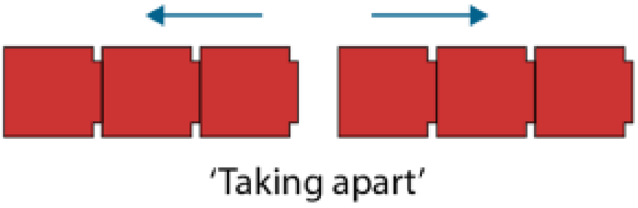
2. Introduction of the- symbol.

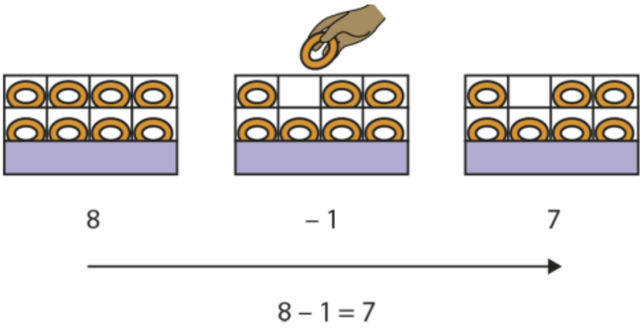
Language:

minuend – subtrahend = difference

- is referred to as subtract or take away

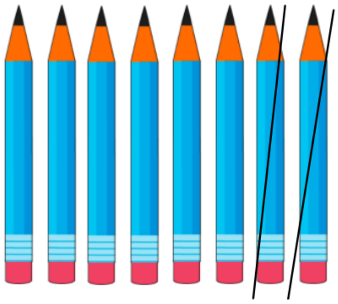
Subtraction means taking apart.





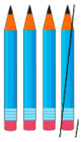
3. Take away objects and count the remaining objects

8 – 2 = 6

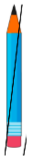


4. The minuend and the subtrahend can not be swapped around

minuend – subtrahend = difference



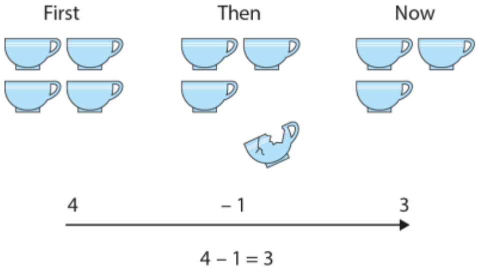
4 – 1 = 3

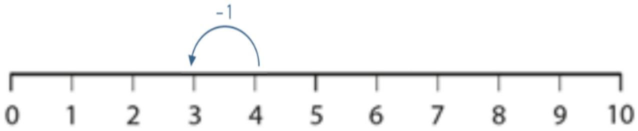


1 – 4 = X

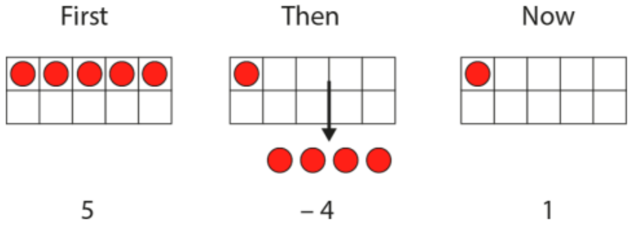
5. Subtracting one gives one less.

Concrete representations will be used to show that, when one item is taken away from the whole, the number shrinks by one, it is one less. Numberlines will be used predominantly as a written calculation for -1.





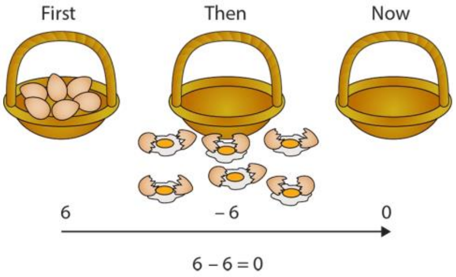
6. Consecutive numbers (numbers one after the other in the counting system) have a difference of one.



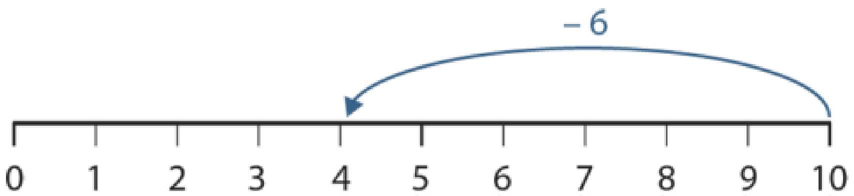
3 + 1	'one more'
1 + 3	
4 – 1	'one less'
4 – 3	'difference of one'

7 When zero is subtracted, the number remains the same. When a number is subtracted from itself, the difference is zero.

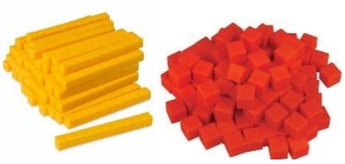
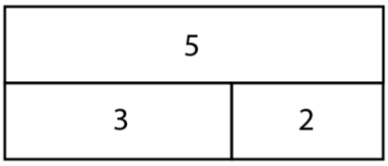
6 + 0	'no more'
0 + 6	
6 – 0	'no less'
6 – 6	'difference of zero'



8. Subtracting 2 or more when the subtrahend (whole) is not bigger than 10) using a numberline.



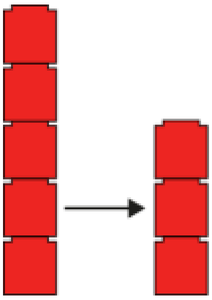
9. Introduction of the following manipulatives (resources):

Diennes	Bar model	Bead string to 100
		

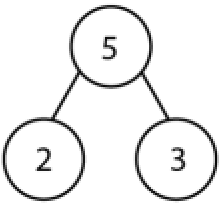
10. Use known facts to deduce others.

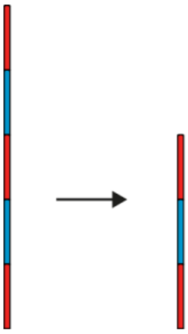
5 – 2 = 3

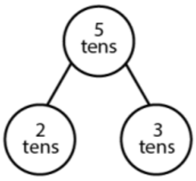
50 – 20 = 30

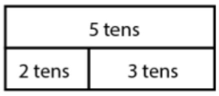


5 – 2 = 3



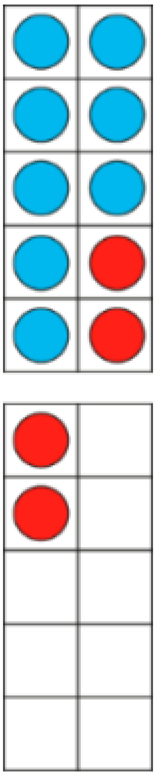


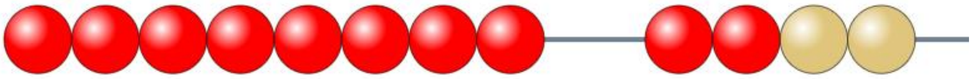


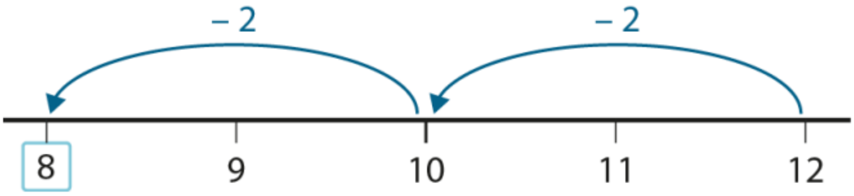


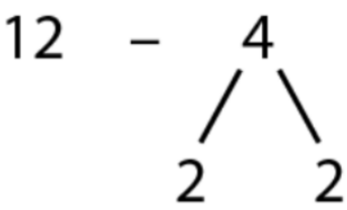
11. The ‘make ten’ strategy for bridging 10 when subtracting

12 – 4 =









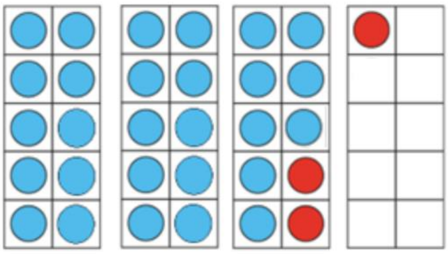
$= 12 - 2 - 2$

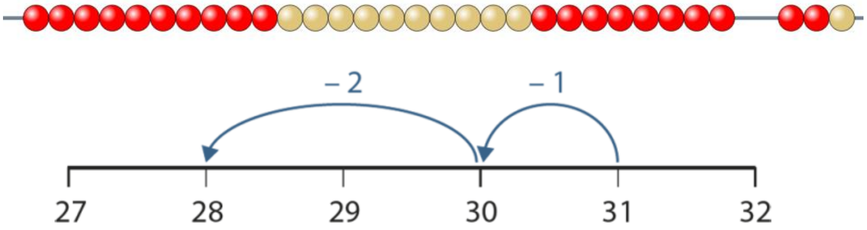
$= 10 - 2$

$= 8$

13. The ‘make ten’ strategy for bridging a multiple of 10.

37 + 5 =





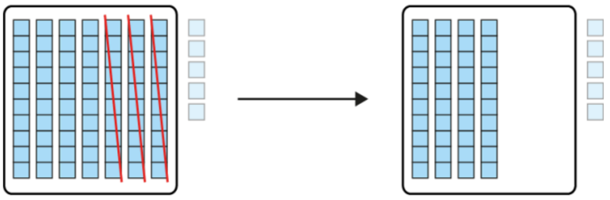
31 – 3 = 31 – 1 – 2

1 2

= 30 – 2

= 28

14. Partitioning to subtract a multiple of 10 from a 2 digit number.



75 - 30 =

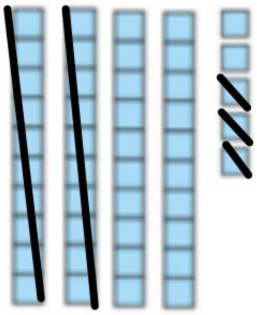
70 5

70 - 30 = 40

40 - 5 = 35

15. Partitioning to subtract two 2 digit numbers (no bridging)

45 – 23 =



45 – 23 = 45 - 20 - 3

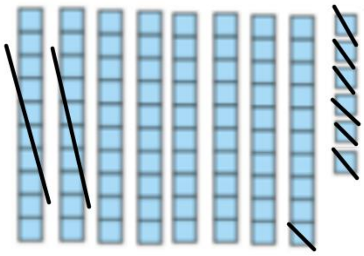
20 3

= 25 - 3

= 22

16. Partitioning to subtract two 2 digit numbers (bridging the tens)

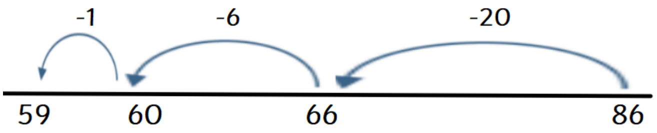
86 – 27 =



86 – 27 =

20 7

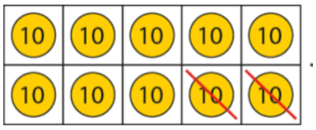
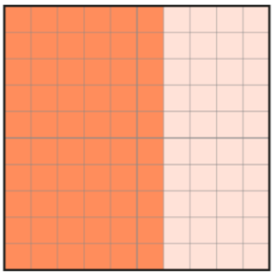
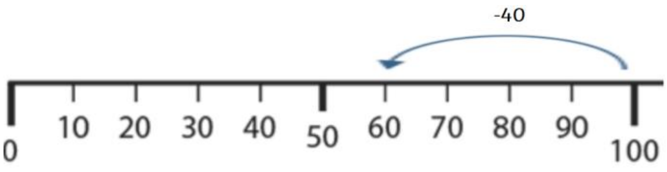
6 1



Introduced in KS2

1. Subtraction within 100 of multiples of 10.

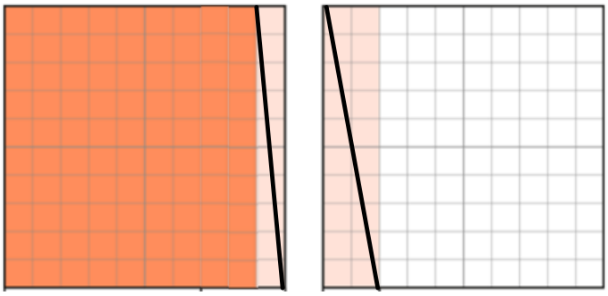
The following manipulatives should be used to 'illuminate' the structure of these calculations.

Tens frames and place value counters.	Hundred grid:	Numberline:
 100 – 20 = 80	 100 – 40 = 60	

2. Subtraction across 100 in multiples of ten- bridging

70 + 50 =

Hundred grids:

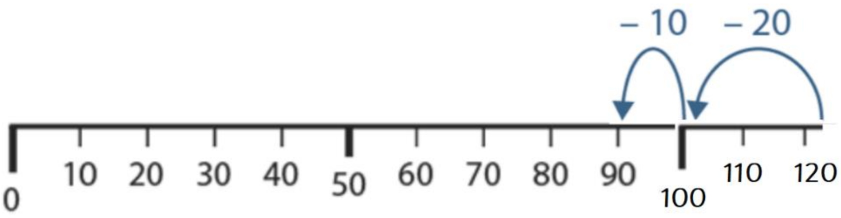


12	–	3	=	9
12 tens	–	3 tens	=	9 tens
120	–	30	=	90

$$\underset{20 \quad 10}{120 - 30} = \boxed{120 - 20} - 10$$

$$= 100 - 10$$

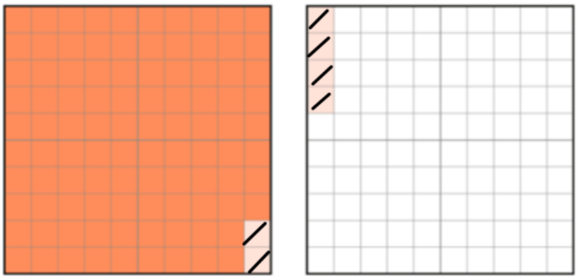
$$= 90$$



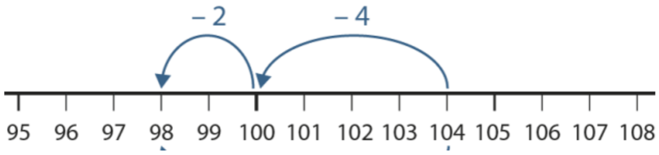
3. Subtraction across 100 – bridging when adding a single digit number.

104 – 6

Hundred grids:



Number line:



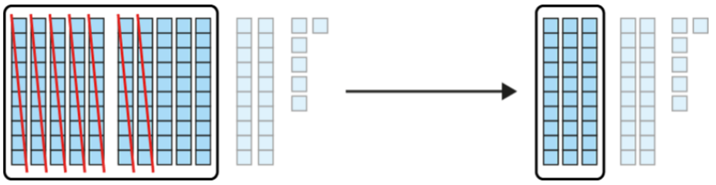
$$\underset{4 \quad 2}{104 - 6} = 104 - 4 - 2$$

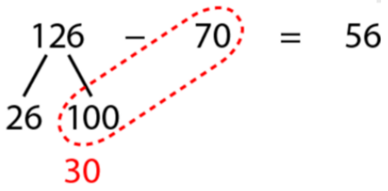
$$= 100 - 2$$

$$= 98$$

4. Subtraction across 100 – when subtracting a multiple of 10.

126 – 70 =

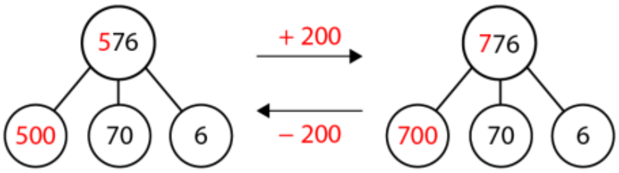




126 – 70 = 100 – 70 + 26
/ \
100 26
= 30 + 26
= 56

5. Addition and subtraction of 10s and 100s to a 3 digit number.

Addition and subtraction of 100s:



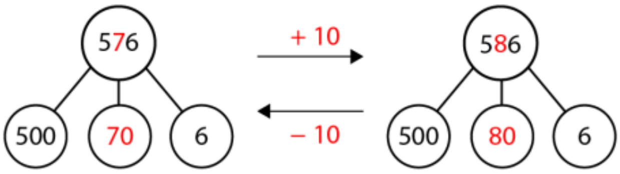
576 + **2**00 = **7**76

776 – **2**00 = **5**76

Example description – addition:

- *'We had* ***five*** *hundreds, then we added* ***two*** *hundreds, so now we have* ***seven*** *hundreds.'*
- ***'Five*** *hundred and seventy-six plus* ***two*** *hundred is equal to* ***seven*** *hundred and seventy-six.'*

Addition and subtraction of tens:



5**7**6 + **1**0 = 5**8**6

5**8**6 – **1**0 = 5**7**6

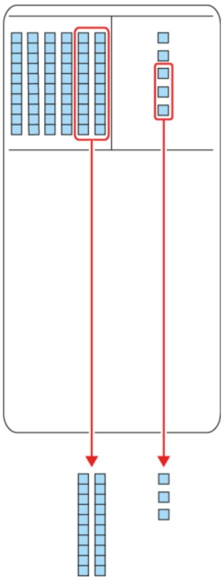
Example description – subtraction:

- *'We had* ***eight*** *tens, then we subtracted* ***one*** *ten, so now we have* ***seven*** *tens.'*
- *'Five hundred and* ***eighty****-six minus* ***ten*** *is equal to five hundred and* ***seventy****-six.'*

6. Applying knowledge of bridging 100:

428 - 50 = 400 - 50 + 28
/ \
400 28
= 350 + 28
= 378

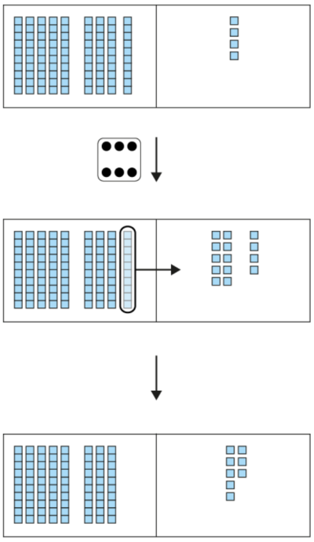
7. A formal method for subtraction – column subtraction – no regrouping



	10s	1s
	6	5
–	2	3
	4	2

8. A formal method for subtraction – column subtraction – with an exchange (exchanging one ten for 10 ones).

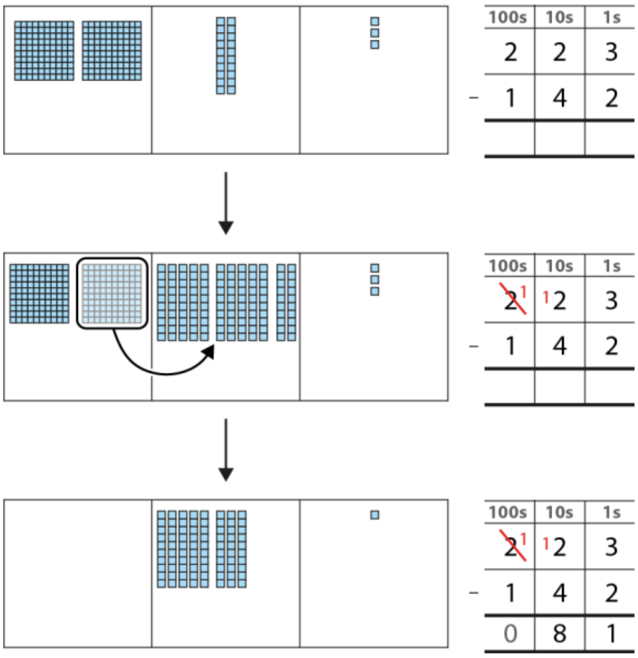
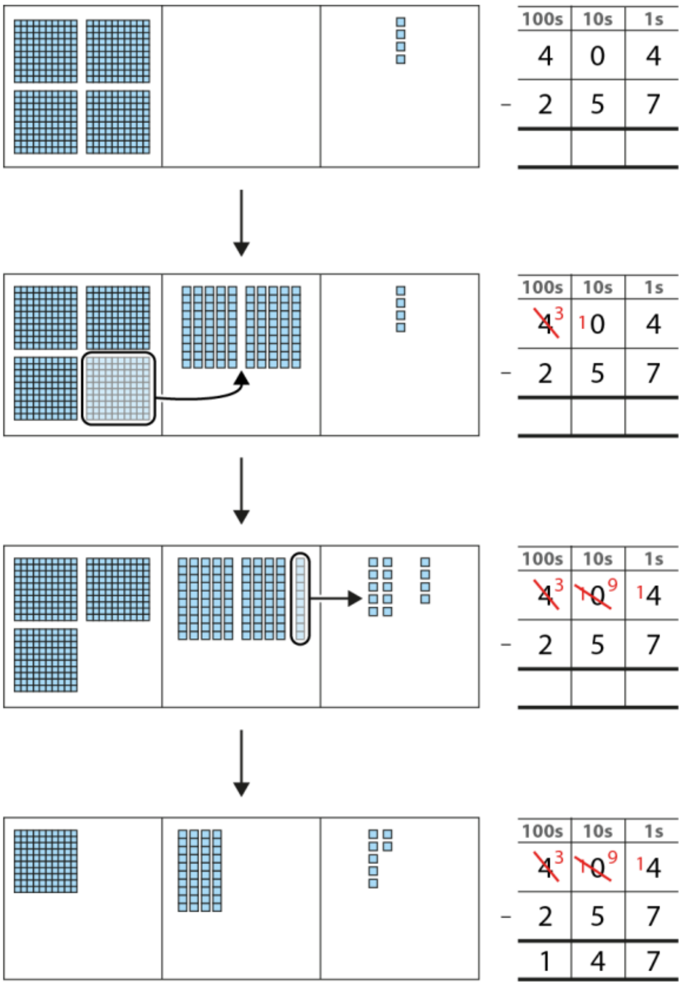
94 – 6



	10s	1s
	9	4
–		6

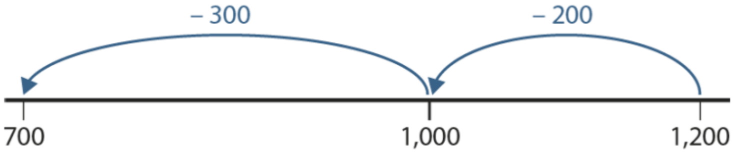
	10s	1s
	$\cancel{9}^{8}$	$^{1}4$
–		6

	10s	1s
	$\cancel{9}^{8}$	$^{1}4$
–		6
	8	8

Column subtraction with 1 exchange	Column subtraction with 2 exchanges
	

9. Subtraction across 1000 in multiples of hundred- bridging

1200 – 500 =



$$\begin{aligned} 1200 - \underset{200 \quad 300}{500} &= \boxed{1200 - 200} - 300 \\ &= 1000 - 300 \\ &= 700 \end{aligned}$$

10. Column subtraction with 4 digit numbers.

A similar progression of concrete, pictorial, abstract would be followed as in point 8 of KS2 subtraction.

No exchange

$$\begin{array}{rrrrr} & 4 & 9 & 9 & 9 \\ - & 4 & 3 & 5 & 8 \\ \hline & & 6 & 4 & 1 \\ \hline \end{array}$$

1 exchange

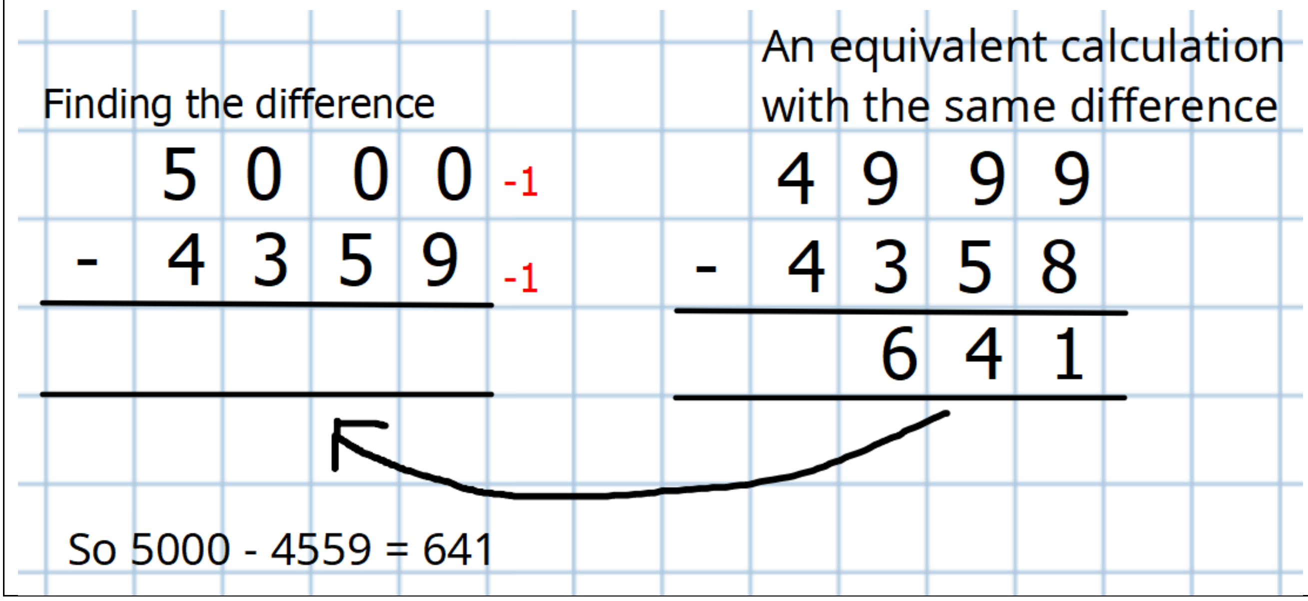
$$\begin{array}{rrrrr} & 4 & 9 & \cancel{9}^{8} & {}^{1}0 \\ - & 4 & 3 & 5 & 8 \\ \hline & & 6 & 3 & 2 \\ \hline \end{array}$$

2 or more exchanges

$$\begin{array}{rrrrr} & 4 & \cancel{9}^{8} & {}^{1}\cancel{3}^{2} & {}^{1}0 \\ - & 4 & 3 & 5 & 8 \\ \hline & & 5 & 7 & 2 \\ \hline \end{array}$$

6. The following mental methods for subtraction will be introduced in Year 4 to support efficient methods.

Finding the difference when calculating with a subtrahend that would involve many exchanges.

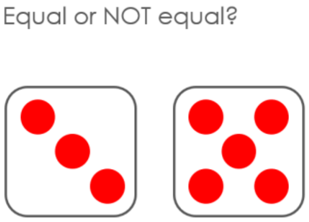


Progression of strategies for Multiplication

Introduced in EYFS and KS1

1. The use of the language **‘equal and unequal’.**

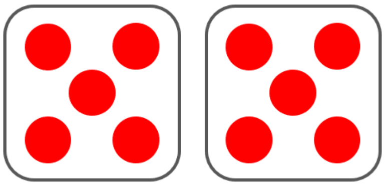
‘Equal’ and ‘unequal’ are used throughout multiplication and division. The language and concept of this is introduced in EYFS and continued into Year 1



“They are not equal because 3 is a part and 5 is a part.”

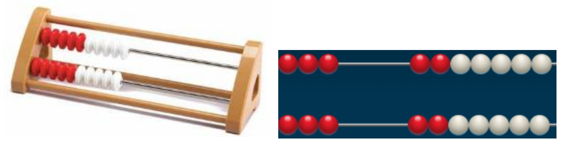
2. The language of **‘double’.**

A double is when 2 equal parts are put together.

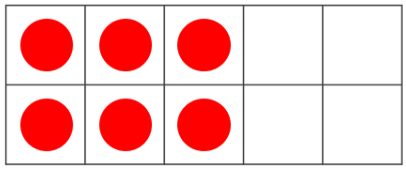


“This is a double because 5 is a part and 5 is a part.”

3. Manipulatives to be introduced – rekenrek and tens frame



3 and 3 are represented on the top and bottom bars of the rekenrek.
“6 is made from 3 and 3.
3 and 3 make 6.”



“6 is made from 3 and 3.
3 and 3 make 6.”

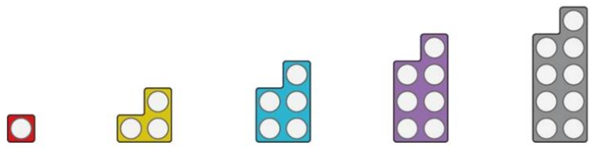
4. Odd and even numbers

Children explore odd and even numbers. Numbers that can be made out of groups of two are even numbers; numbers that can’t be made out of two are odd numbers.

Even numbers:

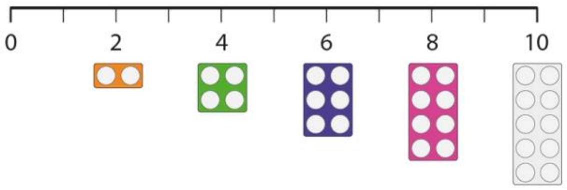
“6 is made of pairs. It is an even number.”

Odd numbers:

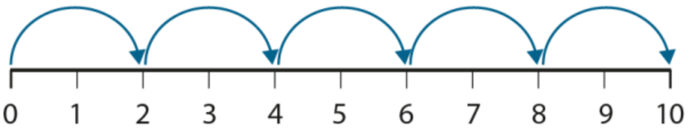


“7 is not made of pairs. It is an odd number.”

At this stage, it is important for children to develop fluency with counting patterns by counting in even patterns.



‘Skip counting’ – highlighting ‘jumps’ of two:

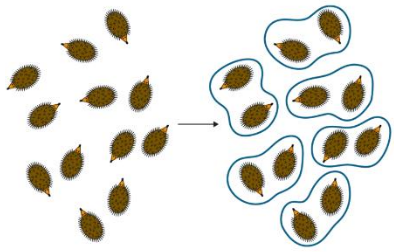


5. Counting in **‘groups of’** 2.

Introduce the children to the language of **‘groups of’.**

How many socks are there? Count in groups of 2.

6. Using counting in **‘groups of 2’** to count an amount where the pairs are not represented.



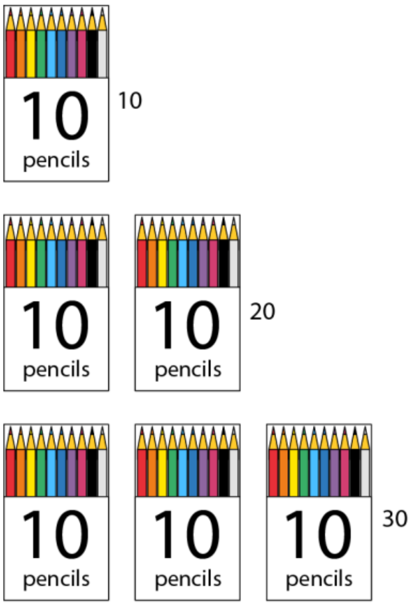
7. Introduction to skip counting in any amount and the vocabulary of ‘multiples of’.

Within Year 1, the children are introduced to counting in groups of 2, 5 and 10. The vocabulary **‘multiples of’** is introduced in Year 1.

- Procedural counting (chanting) alongside real life objects. E.g zero, ten, twenty, thirty… The children will also link back to their counting in groups of. E.g. no tens, one ten, two tens, three tens…

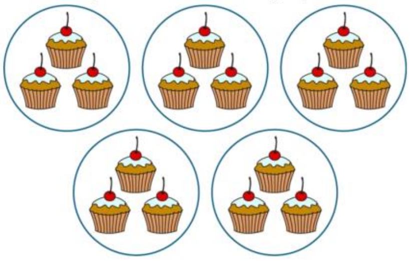
The following manipulatives are useful for learning how to count in groups of:
- Objects – fingers, bags of that amount of items, packs of pens etc.
- Diennes, multilink cubes
- Money
- Measures (e.g using the scale on a thermometer counting up in 10s).
- Scoring in a game (e.g. I score 5 points every time I win a game).

8. Pictorial images should be used wherever possible alongside the skip counting as each new number appears (dual counting). This is so children can see, for example, that three groups of ten and the numeral 30 can sometimes be called ‘thirty’ and sometimes called ‘three tens’.



The same steps and procedures as above are then used within the following year groups for skip counting in different amounts:
Year 2: counting in 3s
Year 3: counting in 4s and 8s
Year 4: counting in 6s, 7s and 9s.

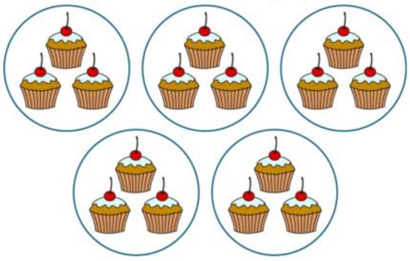
9. Describing equal ‘groups of’



“There’s one group of three, two groups of three, three groups of three, four groups of three, five groups of three.”

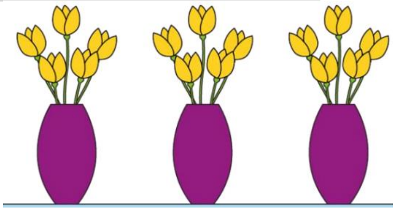
“There are 5 equal groups of 3 cupcakes.
There are 3 cupcakes in each group.
There are 5 groups of 3.”

10. Representing equal groups using repeated addition.



“There are 3 and 3 and 3 and 3 and 3.
We can write this as 3 plus 3 plus 3 plus 3 plus 3.
3 + 3 + 3 + 3 + 3.”

11. Introduction of the multiplication symbol x. Equal groups can be represented with a multiplication expression (symbol).



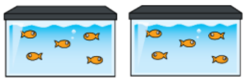
“5 + 5 + 5.
There are 3 groups of 5.
x = groups of.

There are 3 x 5.”

12. Multiplication can be done in any order. It is commutative.

5 x 2

There are 5 tanks with 2 fish in each.

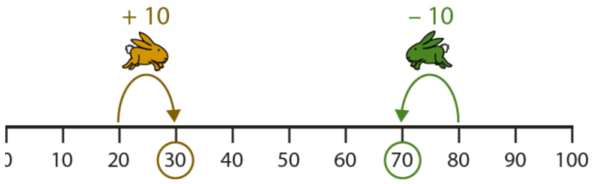


2 x 5

There are 2 tanks with 5 fish in each.

They both have 10 fish altogether.
2 x 5 is the same as 5 x 2
2 x 5 = 5 x 2

13. Use known facts to deduce others.



“If 2 x 10 is 20 then 3 x 10 is one ten more.
If 8 x 10 is 80 then 7 x 10 is one ten less.”

15. Introduction to the vocabulary ‘factors and products’.

Factors = the numbers that we are multiply.
Product = the result of multiplying two whole numbers.

factor x factor = product
5 x 2 = 10
5 is a factor.
2 is a factor.
10 is the product.
The product of 5 and 2 is 10.

16. Use known facts about multiples of and products to deduce others

	These are products in the ten times table. True (✓) or false (✗)?
25, 60, 55, 10, 5	
30, 20, 102, 10, 80, 0, 40	
55, 65, 15, 35, 59	
80, 85, 58, 90, 95, 59	

Which of these numbers could be the product of 17 x 5?

82 84 85

“It must be 85 because multiples of 5 all end in a 5 or 0. All products in the 5x table end in a 5 or 0.”

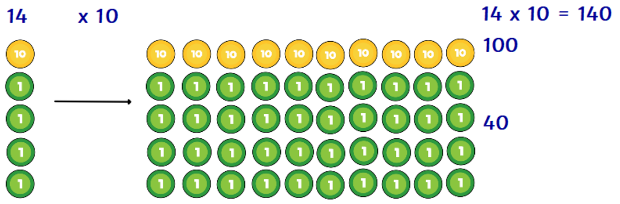
The same steps and procedures as above are then used within the following year groups for each times table.

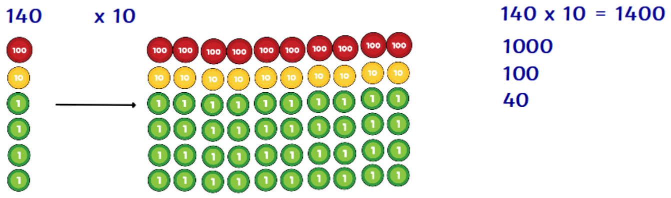
Year 2: 2x, 5x and 10x
Year 3: 3x, 4x, 8x (plus the previous year group times tables)
Year 4: 6x, 7x, 9x, 11x, 12x (plus the previous year group times tables)

17. Using known facts to multiply any number by 10 or by 100. There is a tendency to teach children to 'add a zero on the end or add two zeros on the end' but it is essential that they understand why. That the value of the number is becoming 10 times or 100 times bigger.

14 x 10 – make 14, ten times

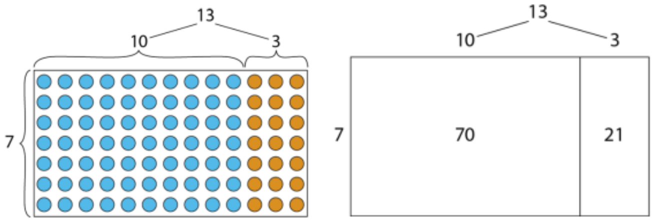
140 x 10 – make 140 ten times.





18. Multiplication by partitioning.

7 x 13



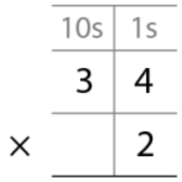
7 x 13 = 7 x 10 + 7 x 3

10 3 = 70 + 21

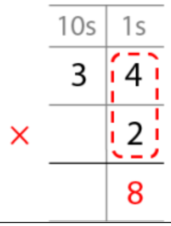
= 91

19. Column multiplication (expanded method).

Step 1 – write the factors:

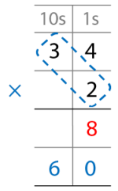


Step 2 – multiply the single-digit number by the ones:



2 × 4 ones = 8 ones

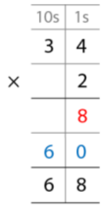
Step 3 – multiply the single-digit number by the tens:



2 × 4 ones = 8 ones

2 × 3 tens = 6 tens

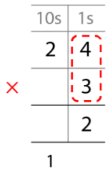
Step 4 – add the partial products:



2 × 4 ones = 8 ones

2 × 3 tens = 6 tens

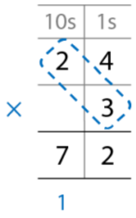
Moving to a compact method – mutliply the second digit number by the ones and regroup.



3 × 4 ones = 12 ones
= 1 ten + 2 ones

'Write "1" below the tens column and "2" in the ones column.'

Multiply the single digit number by the tens and add the tens from regrouping.



3 × 2 tens = 6 tens

6 tens + 1 ten = 7 tens
'Write "7" in the tens column.'

20. HTU x U

521 x3

Multiplication algorithm – expanded layout:

	1,000s	100s	10s	1s
		5	2	1
×				3
				3
			6	0
	1	5	0	0
	1	5	6	3

3 × 1 ones = 3 ones

3 × 2 tens = 6 tens

3 × 5 hundreds = 15 hundreds
= 1 thousand + 5 hundreds

Multiplication algorithm – compact layout:

$$\begin{array}{ccccc} & & 5 & 2 & 1 \\ \times & & & & 3 \\ \hline & 1 & 5 & 6 & 3 \\ \hline \end{array}$$

Multiplication algorithm – expanded layout:

	1,000s	100s	10s	1s
		3	6	7
×				4
			2	8
		2	4	0
	1	2	0	0
	1	4	6	8

4 × 7 ones = 28 ones
= 2 tens + 8 ones

4 × 6 tens = 24 tens
= 2 hundreds + 4 tens

4 × 3 hundreds =12 hundreds
= 1 thousand + 2 hundreds

Multiplication algorithm – compact layout:

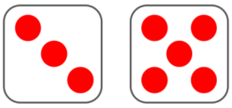
$$\begin{array}{ccccc} & & 3 & 6 & 7 \\ \times & & & & 4 \\ \hline & 1 & 4 & 6 & 8 \\ \hline & & 2 & 2 & \end{array}$$

Progression of strategies for Division

Introduced in EYFS and KS1

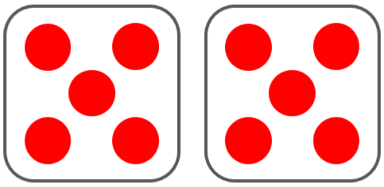
1. The use of the language **‘equal and unequal’.**
‘Equal’ and ‘unequal’ are used throughout multiplication and division. The language and concept of this is introduced in EYFS and continued into Year 1

Equal or NOT equal?



“They are not equal because 3 is a part and 5 is a part.”

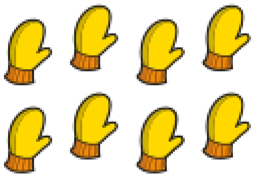
2. The language of **‘half’.**
A half is when a whole amount can be split into two equal parts.



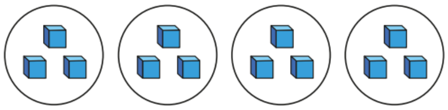
“Half of 10 is 5 because 5 and 5 make 10.

3. Make equal groups- **grouping**
Introduce the children to exploring division through grouping. Start with a given total and make groups of an equal amount. The division symbol is not yet introduced.

Circle groups of 2 mittens and complete the sentence.
There are _______ groups of 2 mittens.



4. Counting how many equal groups there are..

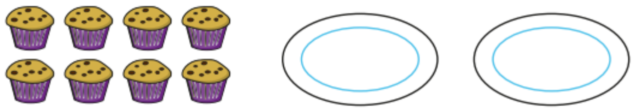


There are 12 cubes altogether.
There are 4 equal groups.
There are 3 cubes in each of those groups.

5. Make equal groups- **sharing**

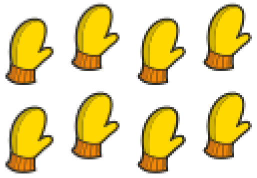
Start with a given amount of an object and physically share it out between groups.

Share the muffins equally between the 2 plates.



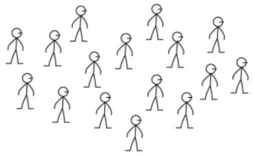
There are ___ muffins altogether.
They are shared equally between ___ plates
There are _____ muffins on each plate.

6. Introduce the words **divided/ division/ dividing**
Linking to the language of *groups of* and *sharing,* the action is now linked to **dividing** in preparation for dividing.



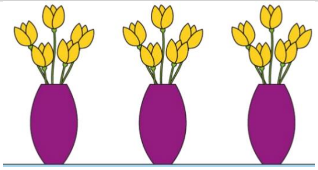
“8 is divided into groups of 2.
There are ___ groups.”

7. Introduce the words **division symbol ÷**
The dividing symbol represents – ‘Putting into groups of’ or ‘sharing between groups’.



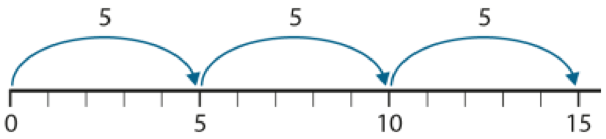
There are 16 children. The children get into pairs.
How many pairs will there be?
We can represent this as 16 ÷ __ (16 into groups of 2).

8. Linking the division symbol to skip counting.



There are 15 flowers. They have been divided into groups of 5.

“One group of 5 is 5, two groups of 5 is 10, three groups of 5 is 15. Fifteen divided into groups of 5 is equal to three.”



9. Introduce the language of division:

30	÷	5	=	6
dividend	÷	divisor	=	quotient

The same steps and procedures as above are then used within the following year groups for dividing by different amounts:

Year 2: divide by 2, 5 and 10
Year 3: divide by 3, 4 and 8
Year 4: divide by 6, 7, 9, 11, 12

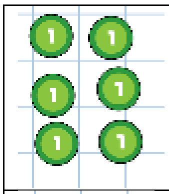
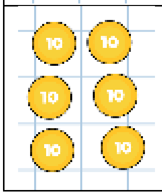
10. Multiplication and division are the inverse (opposite) of each other.

Multiplication can be done in any order (factor x factor = product). Multiplication is commutative.
Division cannot be done in any order (dividend ÷ divisor = quotient.

5 x 2 = 10 10 ÷ 2 = 5
2 x 5 = 10 2 ÷ 10 = not the same answer

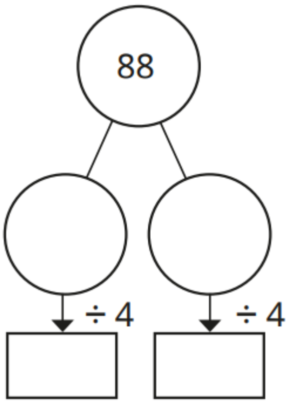
11. Introduction to short division

Known division facts are linked to facts that are tens times bigger

	6 ÷ 2 = 3
	60 ÷ 2 = 30

12. Dividing a two-digit number by a one digit number using partitioning.

88 ÷ 4 = ______



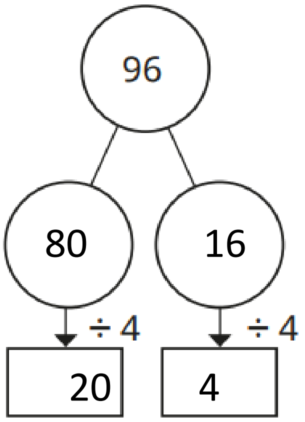
88 is partitioned into an 80 and an 8.

80 ÷ 4 = 20
8 ÷ 4 = 2

88 ÷ 4 = 22

13. Using flexible partitioning to divide a two digit number by a one digit number.

Flexibly partition until multiples of the divisors are reached:
90 and 6
80 and 16 – 80 is divisible by 4 as is 16.



96 ÷ 4 = 24

14. Linking division by partitioning to short division:

84	÷	4	=	21
dividend	÷	divisor	=	quotient

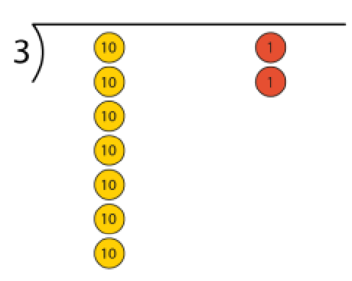
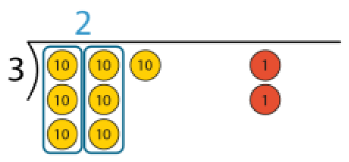
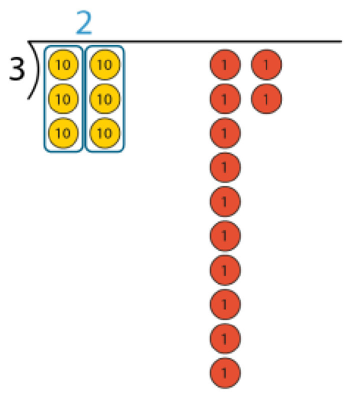
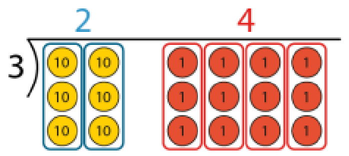
$$\begin{array}{r} 2\;\;1 \\ 4\overline{)8\;\;4} \end{array}$$

$$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\text{dividend}} \end{array}$$

$$\begin{array}{r} 2\;1 \\ 4\overline{)8\;4} \end{array}$$

80 divided by 4 is 20. Write 2 in the tens column.
4 divided by 4 is 1. Write 1 in the ones column.

15. Linking division by flexible partitioning to short division:

Step 1 – write the divisor and dividend	Step 2 – sharing the tens…
 $3\overline{)7\;\;2}$	 $\begin{array}{r} 2\;\; \\ 3\overline{)7\;\;2} \end{array}$
'Seventy-two divided by three.'	7 tens ÷ 3 = 2 tens r 1 ten *'Write "2" in the tens column…'*
Step 3 – …and exchanging	**Step 4 – sharing the ones**
 $\begin{array}{r} 2\;\;\; \\ 3\overline{)7\;\;{}^{1}2} \end{array}$	 $\begin{array}{r} 2\;\;4 \\ 3\overline{)7\;\;{}^{1}2} \end{array}$
1 ten = 10 ones *'…and write "1" to the left of the ones digit of the dividend to make twelve ones.'*	12 ones ÷ 3 = 4 ones *'Write "4" in the ones column.'*