

## INTENT:

At Springdale First School we aim to provide high-quality mathematics education; to provide depth, variety and enjoyment each day in every class; and to provide the optimum environment for all children to succeed.

We aim to deliver consistency in pedagogy: it is crucial for children to develop procedural fluency alongside a deep grasp of the base ten number system which stems from all staff teaching the right thing at the right time.

We aim to enable aspiration: secure mathematical skills underpin success in everyday life and facilitate the pursuit of fulfilling lives and well-paid careers.

## Context:

The aims of the National Curriculum (2014) states that all children should progress at broadly the same rate whilst developing fluency in fundamental number work, reasoning abilities (both in terms of language and mathematical proof) and the capacity to solve problems (demonstrating flexible and creative thinking in a range of situations).

## Mastery:

Maths is a mastery subject. At Springdale this means we believe that all children are capable of understanding and doing mathematics, given the appropriate resources and sufficient time. Children need to build solid foundations so that their mathematical ability broadens and deepens over time: this requires looking at concepts in detail using a variety of representations and contexts and committing key facts, such as number bonds and times tables, to memory. Opportunities to acquire and practise mathematical knowledge need to be carefully designed to deliver rich experiences within which calculation algorithms are taught consistently throughout the school and across year groups. These then need to be constantly revisited to ensure they are committed to long term memory and can be easily recalled when needed.

Guidance on using the Springdale Maths Calculation Policy

- While these methods are linked to year groups, this should only be as a guide and each year group should
assess children's knowledge and understanding of concepts. Methods should be built upon and revisited by every year group to ensure retention of each method.
- Teaching written calculation methods does not replace the need to revisit and use mental strategies. These are the building blocks of solid number work.
- All the images selected should act as guidance and not constraints.
- Children should be encouraged to first approximate their answers before calculating, and then check their answers after calculating using an appropriate strategy.
- Children should be encouraged to consider if a mental calculation would be more appropriate before using a written method.

This policy will ensure consistency and progression in our approach to the learning and teaching of calculations across different year groups. It will enable our children, teachers and parents to work in partnership, developing an efficient, reliable, formal written method of calculation for all operations and to use these methods accurately with confidence for understanding.

It is important to ensure that everyone who uses this policy understands that the methods described under each year group are not specific to that year group. Each method should be recalled, utilised, and built upon in a way that is suited to the calculations and content being taught.

## IMPLEMENTATION:

At Springdale, Maths lessons are fun, practical learning opportunities that take place both inside and outside the classroom. They involve lots of discussion and investigation before learning and practising and then moving onto applying.
Children are taught reasoning and problem solving skills to ensure an indepth understanding of each concept.
Within our curriculum, time is prioritised to improve number sense and arithmetic proficiency. After developing fluency, children need to show that they can apply their knowledge in mathematics. Therefore, for all maths concepts teachers need to ensure that children are "challenged through being offered rich and sophisticated
problems."
We aim to do this through the development of conceptual and procedural variation:

- Procedural variation provides step-by-step intelligent practice, looking at what has stayed the same and what has changed. Certain aspects vary while others are kept constant. The ability to recall facts and manipulate them to work out other facts is important so it involves practice to spot relationships and make connections.
- Conceptual variation explores what something is and isn't, and what's the same or different. This time, instead of varying the problem, the representation of the problem is varied. This is generally through the use of models or images, as well as application across a range of contexts (e.g. time, money, measurement).


## Planning

We believe that students who have a good grasp of number make better mathematicians. Spending longer on mastering key skills will build a student's confidence and help develop a secure understanding. Therefore, the Autumn term's planning has a primary focus on developing number sense, place value and arithmetic skills across the school. We use 'Big Maths' calculation assessments as well as end of unit independent assessments that are spaced out to assess children's level of attainment. We also use end of term NTS assessments inform our planning and identify any areas of number work that need greater attention and revisiting.

- Planning is based on the programmes of study in the new curriculum, using published materials (White Rose scheme of work) as guidance. Teachers select their focused key objectives from the National Curriculum.
- In EYFS, maths activities are accessible at all times during child initiated learning.

To ensure key concepts are embedded into long term memory, '5-a-day' books are used which primarily focus on number. By constantly revisiting these key concepts, students become more secure which mathematical procedures and concepts. We understand that some children need more revisiting of some concepts than others. For this, we have 'Keep Up, Catch Up' sessions with teaching assistants. We also use Precision Teaching to help
build speed and fluency.

## Key Number Facts

At Springdale First School we recognise the importance of establishing a secure foundation in mental calculations and recall of number facts before standard written methods are introduced. This begins with regular and frequent counting. For example:

- Reception: on and back in ones
- Y1 counting in $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s
- Y2 make links with counting in 10s crossing boundaries
- Y3 begin to apply this knowledge to reading scales and fractions (counting in tenths - 0.1, 0.2, 0.3, etc. and halves $0.5,1.0,1.5,2.0$, etc. quarters $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1,1 \frac{1}{4}$ etc.)
- Y4 count using a range of multiples and in a number of ways (e.g. 25s, 250s, 2.5s, 0.25s)

We use materials such as 'TimesTables Rockstars' and 'NumBots' to provide regular practise of key number facts to develop fluency. We aim for students to practise counting, subitising, and times table facts in school at 3-4 times a week and at home we encourage them to practise for 20 minutes per week minimum.

## Timestables to Learn:

Year 2 - 2, 5 and 10
Year 3 - as above and 3, 4 and 8 These should be achieved in a minimum of 6 seconds or less.
Year 4 - all times tables to $12 \times 12$
Alongside this, we have also created a 'Maths Rockstars' programme that focuses on key addition, multiplication, and division facts. These are broken into 'steps'. Students start learning and being tested on each step from the Summer term in Year 1.

When being tested, students are allowed 3 minutes per Maths Rockstar sheet. If they complete the sheet in the allotted time, achieving 100\%, then they will progress to the next step.

Year 1 - Achieving to Step 6
Year 2 - Achieving to Step 12
Year 3 - Achieving to Step 18
Year 4 - Achieving to Step 28

## Concrete, pictorial, abstract

We begin our maths teaching with practical resources wherever possible. Concrete apparatus supports children with their ability to grasp new concepts. We then move onto pictorial representations and 'bar models' to represent numbers in worded problems, fractions and, later, algebra. This helps our pupils to visualise the relationships between numbers.

## Cross-curricular links

Mathematics is mainly taught as a discrete subject but every effort is made to link maths with other areas of the curriculum. We try and identify the mathematical possibilities across the curriculum at the planning stage. We also draw children's attention to the links between maths and other curricular work so that they begin to understand that maths is not an isolated subject. In the Early Years, these links are more evident through play based learning.

## Learning environment

We recognise the importance of displays in the teaching and learning of mathematics. Every class displays a working wall which demonstrates examples of today's lesson as well as previous lessons. This is because learning is a continuum and prior learning should be recalled and retrieved before moving onto something new.

Displays will also include key vocabulary for children to call upon when working independently, especially in relation to reasoning questions. Also available should be examples of bar models and strategies for problemsolving (use of tables, drawing pictures, trial and improve etc.). Bu having these readily available to students, it will foster a greater sense of independence when working.

## Counting

We also understand the importance of regularly counting practice. Students have many opportunities to count forwards and backwards from different starting points to see patterns and understand the mechanics of a base-10 number system. Each year group will also count forwards and backwards in different jumps, depending on the knowledge relevant.

When counting, students will be exposed to a variety of resources, both physical and online. This includes hundred squares, number lines, number tracks, dienes, numicon, and place value flip chart (online). Discussions around what is happening and why take place and allow students to predict which number will appear next.

## Subitising

We understand the importance of understanding what number is, 'The Threeness of Three'. Students are constantly taught subitising skills to ensure they gain a solid understanding of what a number is and what it is made up of. This can include the use of numicon, tens frame, dots, playing cards, pictures etc.


$$
\begin{array}{ll}
7=4+3 & 7-3=4 \\
7=3+4 & 7-4=3
\end{array}
$$




## Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

## Bar

## Concrete



## 0000000

## Discrete



7
Combination
4


## Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use bar models to represent larger numbers, decimals and fractions.

## Bar Model - Comparison

## Discrete



$$
7-3=4
$$

## Continuous


$7-3=4$
$2,394-1,014=1,380$

## Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

## Number Shapes - Numicon



## Benefits

Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1 , they can see that the other number decreases by 1 to find all the possible number bonds for a number.

$7=3+4$

$7-3=4$

$7-3=4$

## Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

$$
\begin{aligned}
& 4+3=7 \\
& 3+4=7 \\
& 7-3=4 \\
& 7-4=3
\end{aligned}
$$

## 4 is a part.

 3 is a part. 7 is the whole.
## Benefits

When adding and subtracting within 10 , the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.


$$
7+6+3=16
$$



## Benefits

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10 , and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10 , this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

## -00-00000000--000-0000000-

## -00-0000000000000000000--000-00000000000000000-



## Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10 .
They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. $2+8=10$, move one bead, $3+7=10$.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20 .

Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.
$5+3=8$


$$
10-4=6
$$

| 1 | 2 | 3 | 4 | 5 | 0 | 7 | 8 | 9 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

$8+7=15$
ハைை


## Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.
$5+3=8$


## Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.
$35+37=72$

$35+37=72$

$72-35=37$


## Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.
$7+6=13$

$42-17=25$

## bundle together groups of 10



## Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.


## Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange.. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children. How many ones are there altogether?
Can we make an exchange? (Yes or No)
How many do we exchange? ( 10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column) How many ones do we have left? (Write in ones column) Repeat for each column.

## Base <br> Ten / <br> Dienes



| Hundreds | Tens | Ones | 3135 |
| :---: | :---: | :---: | :---: |
|  |  | -4ty | $-273$ |
|  |  |  | 262 |

## Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.
This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

## Place Value Counters


3.65


## Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

## Place Value Chart

| Hundreds | Tens | Ones |
| :--- | :--- | :--- |
| $9000 \phi$ | $0000 \phi$ | 00 |
| $\varnothing$ |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

$$
\begin{array}{r}
652 \\
-207 \\
\hline 445 \\
\hline
\end{array}
$$



## Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

## Addition

| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Add two 1-digit <br> numbers to 10 | 1 | Part-whole model <br> Bar model <br> Number shapes | Ten frames (within 10) <br> Bead strings (10) <br> Number tracks |
| Add 1 and 2-digit <br> numbers to 20 | 1 | Part-whole model <br> Bar model <br> Number shapes <br> Ten frames (within 20) | Bead strings (20) <br> Number tracks <br> Number lines (labelled) <br> Straws |
| Add three 1-digit <br> numbers | 2 | Part-whole model <br> Bar model | Ten frames (within 20) <br> Number shapes |
| Add 1 and 2-digit <br> numbers to 100 | 2 | Part-whole model <br> Bar model <br> Number lines (labelled) | Number lines (blank) |
| Straws |  |  |  |
| Hundred square |  |  |  |


| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Add two 2-digit <br> numbers | 2 | Part-whole model <br> Bar model <br> Number lines (blank) <br> Straws | Base 10 <br> Place value counters <br> Column addition |
| Add with up to 3-digits | 3 | Part-whole model <br> Bar model | Base 10 <br> Place value counters <br> Column addition |
| Add with up to 4-digits | 4 | Part-whole model <br> Bar model | Base 10 <br> Place value counters <br> Column addition |

Skill: Add 1-digit numbers within 10 Year: 1



| Skill: Add 1-digit and 2-digit numbers to 100 |  |  |  |  |  |  |  |  |  |  |  | Year: 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  <br> 38 <br> $38+5=43$ |  |  |  |  |  |  |  |  |  |  |  | When adding single digits to a two-digit number, children should be encouraged to count on from the larger number. |
|  |  |  |  |  |  |  |  |  |  |  |  | y their knowledge |
|  |  |  |  | 2 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | 22. | 2324 |  | ${ }^{26}$ | ${ }^{27}$ |  |  |  | $\begin{aligned} & \text { e.g. } 8+5=13 \text { so } 38 \\ & +5=43 . \end{aligned}$ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | 5253 | 5354 |  | 5 |  |  |  |  |  |
|  |  |  |  | 6263 | 6364 |  | 66 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  | hildren to find the |
|  |  |  |  |  |  |  |  |  |  |  |  | mber bond to 10 . |





## Subtraction

| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Subtract two 1-digit numbers to 10 | 1 | Part-whole model Bar model Number shapes | Ten frames (within 10) Bead strings (10) Number tracks |
| Subtract 1 and 2-digit numbers to 20 | 1 | Part-whole model <br> Bar model <br> Number shapes <br> Ten frames (within 20) | Bead string (20) <br> Number tracks Number lines (labelled) Straws |
| Subtract 1 and 2-digit numbers to 100 | 2 | Part-whole model Bar model Number lines (labelled) | Number lines (blank) Straws Hundred square |
| Subtract two 2-digit numbers | 2 | Part-whole model Bar model Number lines (blank) Straws | Base 10 <br> Place value counters Column addition |


| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Subtract with up to 3- <br> digits | 3 | Part-whole model <br> Bar model | Base 10 <br> Place value counters <br> Column addition |
| Subtract with up to 4- <br> digits | 4 | Part-whole model <br> Bar model | Base 10 <br> Place value counters <br> Column addition |


Skill: Subtract 1 and 2-digit numbers to 20 Year: 1




## Glossary - Addition and Subtraction

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative - numbers can be added in any order.
Complement - in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference - the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange - Change a number or expression for another of an equal value.

Minuend - A quantity or number from which another is subtracted.

Partitioning - Splitting a number into its component parts.

Reduction - Subtraction as take away.
Subitise - Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total - The aggregate or the sum found by addition.

## Bar Model



Girls


## Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?
The multiple bar model provides an opportunity to compare the groups.

## Number Shapes - Numicon


$5 \times 4=20$
$4 \times 5=20$

## 88888888

$$
18 \div 3=6
$$



## Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd $\times$ odd $=$ even, odd $\times$ even $=$ odd, even $\times$ even $=$ even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18 .

## Bead String

## Benefits

## $-000-000-000-000-000-$

$5 \times 3=15$
$3 \times 5=15$

## $-00000-00000-00000-$

$5 \times 3=15$
$15 \div 5=3$
$3 \times 5=15$
$15 \div 3=5$

$4 \times 5=20$
$20 \div 4=5$
$5 \times 4=20$

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.
Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 - Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

## Number Tracks



$$
\begin{aligned}
& 6 \times 3=18 \\
& 3 \times 6=18
\end{aligned}
$$



$$
18 \div 3=6
$$

## Benefits

Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.
When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0 .
Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

## Number Line (labelled)


$4 \times 5=20$
$5 \times 4=20$

$20 \div 4=5$

## Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.
When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0 .
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

## Number Line (Blank)



A red car travels 3 miles.
A blue car 4 times further. How far does the blue car travel?


A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

## Benefits

Children can use blank number lines to represent scaling as multiplication or division.

Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

## Base 10/ Dienes



## Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces This area model can be linked to the grid method or the formal column method of multiplying 2 -digits by 2 -digits.

## Benefits


$68 * 2=34$
Using Base 10 or Dienes is an effective way to support children's understanding of division.

When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

$72: 3=24$


When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the partwhole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

## Place Value Counters



## Benefits

Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2digit numbers by 2 -digit numbers.



## Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.

| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Recall and use multiplication and division facts for the 2-times table | 2 | Bar model Number shapes Counters Money | Ten frames Bead strings Number lines Everyday objects |
| Recall and use multiplication and division facts for the 5-times table | 2 | Bar model Number shapes Counters Money | Ten frames <br> Bead strings <br> Number lines <br> Everyday objects |
| Recall and use multiplication and division facts for the 10-times table | 2 | Hundred square Number shapes Counters Money | Ten frames Bead strings Number lines Base 10 |


| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Recall and use <br> multiplication and <br> division facts for the <br> 3-times table | 3 | Hundred square <br> Number shapes <br> Counters | Bead strings <br> Number lines <br> Everyday objects |
| Recall and use <br> multiplication and <br> division facts for the <br> 4-times table | 3 | Hundred square <br> Number shapes <br> Counters | Bead strings <br> Number lines <br> Everyday objects |
| Recall and use <br> multiplication and <br> division facts for the <br> 8-times table | 3 | Hundred square <br> Number shapes | Bead strings <br> Number tracks <br> Everyday objects |
| Recall and use <br> multiplication and <br> division facts for the <br> 6-times table | 4 | Hundred square <br> Number shapes | Bead strings <br> Number tracks <br> Everyday objects |


| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Recall and use <br> multiplication and <br> division facts for the <br> 7-times table | 4 | Hundred square <br> Number shapes | Bead strings <br> Number lines |
| Recall and use <br> multiplication and <br> division facts for the <br> 9-times table | 4 | Hundred square <br> Number shapes | Bead strings <br> Number lines |
| Recall and use <br> multiplication and <br> division facts for the <br> 11-times table | 4 | Hundred square <br> Base 10 | Place value counters |
| Recall and use <br> multiplication and <br> division facts for the <br> 12-times table | 4 | Number lines |  |
| Handred square | Place value counters |  |  |
| Bumber lines |  |  |  |



| Skill: 5 times table |  |  |  |  |  |  |  |  | Year: 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 <br> 12 <br> 22 <br> 32 <br> 42 | 3 <br> 13 <br> 23 <br> 33 <br> 43 |  | 6  <br> 16  <br> 26  <br> 36  <br> 46  |  | $\begin{array}{\|c\|} \hline 8 \\ \hline 18 \\ \hline 28 \\ \hline 38 \\ \hline 48 \\ \hline \\ \hline \end{array}$ | In <br>  | 1 1 1 1 <br>  5 10 15$-00000-0$ | Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. <br> Look for patterns in the five times table, using concrete manipulatives to support. Notice the pattern in the ones as well as highlighting the odd, even, odd, even pattern. |






| Skill: 6 times table |  |  |  |  |  |  |  |  |  |  |  |  |  |  | Year: 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | 2 | 3 | 4 | 5 | (6) |  |  | 9 | 10 | Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the six times table, using manipulatives to support. Make links to the 3 times table, seeing how each multiple is double the threes. Notice the pattern in the ones within each group of five multiples. <br> Highlight that all the multiples are even using number shapes to support. |
|  |  |  |  |  | 11 | (12) | 13 | 14 | 15 | 16 | 17 | (18) | 19 | 20 |  |
|  |  |  |  |  | 21 | 22 | 23 | (4) | 25 | 26 | 27 | 28 | 29 | (3) |  |
|  |  |  |  |  | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |  |
|  |  |  |  |  | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |  |
|  |  |  |  |  | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |  |
| 6 | 12 | 18 | 24 | 30 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |  |
| 36 | 42 | 48 | 54 | 60 |  | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |  |
| 66 | 72 | 78 | 84 | 90 | 91 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |  |
| $-00000-000-$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |


| Skill: 9 times table |  |  |  |  |  |  |  |  |  |  |  |  |  |  | Year: 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0009000900000000000000$ |  |  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | (9) | 10 | Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the nine times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support as well as noting the odd, even pattern within the multiples. |
|  |  |  |  |  | 11 | 12 | 13 | 14 | 15 | 16 | 17 | (18) | 19 | 20 |  |
|  |  |  |  |  | 21 | 22 | 23 | 24 | 25 | 26 | (7) | 28 | 29 | 30 |  |
|  |  |  |  |  | 31 | 32 | 33 | 34 | 35 | (36) | 37 | 38 | 39 | 40 |  |
|  |  |  |  |  | 41 | 42 | 43 | 44 | (15) | 46 | 47 | 48 | 49 | 50 |  |
| 9 | 18 | 27 | 36 | 45 | 51 | 52 | 53 | (4) | 55 | 56 | 57 | 58 | 59 | 60 |  |
| 54 | 63 | 72 | 81 | 90 | 61 | 62 | (3) | 64 | 65 | 66 | 67 | 68 | 69 | 70 |  |
| 71 72 73 74 75 76 77 78 79 80 <br> 81 82 83 84 85 86 87 88 89 90 <br> 91 92 93 94 95 96 97 98 99 100 <br> -000000000-000000000-000000000 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |





| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Solve one-step <br> problems with <br> multiplication | $1 / 2$ | Bar model <br> Number shapes <br> Counters | Ten frames <br> Bead strings <br> Number lines |
| Multiply 2-digit by 1- <br> digit numbers | $3 / 4$ | Place value counters <br> Base 10 | Short written method <br> Expanded written method |
| Multiply 3-digit by 1- <br> digit numbers | 4 | Place value counters | Base 10 |

Skill: Solve 1-step problems using multiplication $\quad$\begin{tabular}{l}
Year: $1 / 2$ <br>

| Children represent |
| :--- |
| multiplication as |
| repeated addition in |
| many different ways. | <br>

In Year 1, children use <br>
concrete and pictorial <br>
representations to <br>
solve problems. They <br>
are not expected to <br>
record multiplication <br>
formally.
\end{tabular}




## Division

| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Solve one-step <br> problems with division <br> (sharing) | $1 / 2$ | Bar model <br> Real life objects | Arrays <br> Counters |
| Solve one-step <br> problems with division <br> (grouping) | $1 / 2$ | Real life objects <br> Number shapes <br> Bead strings <br> Ten frames | Number lines <br> Arrays <br> Counters |
| Divide 2-digits by 1- <br> digit (no exchange <br> sharing) | 3 | Straws <br> Base 10 <br> Bar model | Place value counters |
| Divide 2-digits by 1- <br> digit (sharing with <br> exchange) | 3 | Straws <br> Base 10 | Place value counters |
| Bar model | Part-whole model |  |  |


| Skill | Year | Representations and models |  |
| :---: | :---: | :---: | :---: |
| Divide 2-digits by 1- <br> digit (sharing with <br> remainders) | $3 / 4$ | Straws <br> Base 10 <br> Bar model | Place value counters <br> Part-whole model |
| Divide 2-digits by 1- <br> digit (grouping) | $3 / 4$ | Place value counters <br> Counters | Place value grid <br> Written short division |

Skill: Solve 1-step problems using multiplication (sharing) $\quad$\begin{tabular}{l}

\multicolumn{1}{|c|}{| Year: $1 / 2$ |
| :--- |} <br>

\hline lhildren solve <br>
problems by sharing <br>
amounts into equal <br>
groups.
\end{tabular}

Skill: Solve 1-step problems using division (grouping) $\quad$| Year: $1 / 2$ |
| :--- |
| lildren solve |
| problems by grouping |
| and counting the |
| number of groups. |
| Grouping encourages |
| children to count in |
| multiples and links to |
| repeated subtraction |
| on a number line. |
| They can use |
| concrete |

Skill: Divide 2-digits by 1-digit (sharing with no exchange) | Year: 2 |
| :--- |
| Tens |

Skill: Divide 2-digits by 1-digit (sharing with exchange) $\quad$\begin{tabular}{l}

\multicolumn{1}{c|}{| Year: $3 / 4$ |
| :--- |} <br>


| When dividing in Year |
| :--- |
| 3 and 4, there should |
| be a switch from |
| division as sharing' to |
| division as 'grouping'. |
| Children should be |
| taught to make a |
| group of the dividend. |
| If dividing numbers |
| involving an |
| exchange, children |
| can use Base 10 and |
| place value counters |
| to exchange one ten |
| for ten ones. | <br>


| Children should start |
| :--- |
| with the equipment |
| and make groups of |
| the dividend starting |
| with the tens before |
| moving onto the ones. | <br>

\hline
\end{tabular}

## Glossary - Multiplication and Division

Array - An ordered collection of counters, cubes or other item in rows and columns.

Commutative - Numbers can be multiplied in any order.

Dividend - In division, the number that is divided.

Divisor - In division, the number by which another is divided.

Exchange - Change a number or expression for another of an equal value.

Factor - A number that multiplies with another to make a product.

Multiplicand - In multiplication, a number to be multiplied by another.

Partitioning - Splitting a number into its component parts.

Product - The result of multiplying one number by another.

Quotient - The result of a division
Remainder - The amount left over after a division when the divisor is not a factor of the dividend.

Scaling - Enlarging or reducing a number by a given amount, called the scale factor

